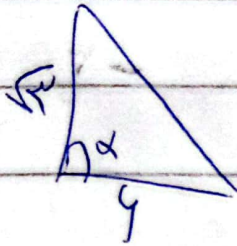


مسألة ٢٠١٥

أبسط

رقم ١٥

(1)



$$\frac{1}{10} \times 9 \times \sqrt{10} \times \sin \alpha = \frac{9 \times 10}{10} = 9$$

$$\sin \alpha = \frac{9}{10 \times \sqrt{10}}$$

$$\sin \alpha = \frac{\sqrt{10}}{10}$$

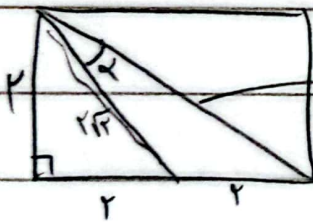
$$\sin \alpha = \frac{\sqrt{10}}{10} \rightarrow \alpha_{\text{max}} = \frac{\pi}{4}$$

$$\sin \alpha = \frac{\sqrt{10}}{10} \rightarrow \alpha_{\text{min}} = \frac{3\pi}{4}$$

$$\frac{\alpha_{\text{max}}}{\alpha_{\text{min}}} = \frac{\frac{\pi}{4}}{\frac{3\pi}{4}} = \frac{1}{3}$$

Cota = ?

(2)



$$\sqrt{1^2 + 1^2} = \sqrt{2}$$

$$c = \sqrt{a^2 + b^2 - 2ab \cos \alpha} \rightarrow c^2 = (1\sqrt{2})^2 + (1\sqrt{2})^2 - 1 \times 1\sqrt{2} \times 1\sqrt{2} \times \cos \alpha$$

$$c^2 = 2 + 2 - 2 \times \cos \alpha$$

$$-2c = -2 \times \cos \alpha$$

$$\frac{2c}{2} = \cos \alpha \rightarrow \cos \alpha = \frac{c}{2}$$

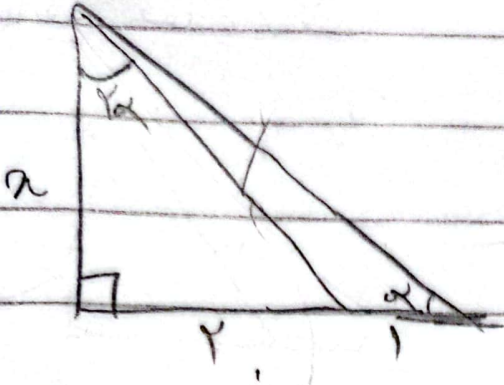
$$\cos^2 \alpha + \sin^2 \alpha = 1 \rightarrow \sin^2 \alpha = 1 - \frac{c^2}{4} = \frac{4 - c^2}{4}$$

$$\sin \alpha = \frac{\sqrt{4 - c^2}}{2}$$

$$\cot \alpha = \frac{\frac{c}{2}}{\frac{\sqrt{4 - c^2}}{2}} = \frac{c}{\sqrt{4 - c^2}}$$



! cot α (μ)



$$\cot \alpha - \tan \alpha = \mu \cot \alpha$$

$$\frac{y}{a} - \frac{a}{y} = \mu \times \frac{a}{y}$$

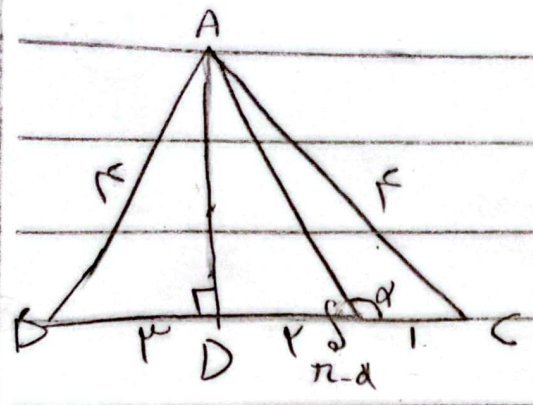
$$\frac{y - a^2}{ay} = \mu \rightarrow y - a^2 = \mu a$$

$$y = \mu a + a^2$$

$$\frac{y}{a} = \mu + a \rightarrow a = \frac{y}{\mu + a}$$

$$\cot \alpha = \frac{y}{\frac{y}{\mu + a}} = \mu + a = \mu + \frac{y}{\mu + a}$$

(2)



$$AD = \sqrt{r^2 - p^2} = \sqrt{14 - 9} = \sqrt{5}$$

$$\tan(\pi - \alpha) = -\tan \alpha = \frac{-\sqrt{5}}{r}$$

(1) $r \sin^p \alpha + \cos^p \alpha = \frac{r}{p}$ (A)

$$r(1 - \cos^p \alpha) + \cos^p \alpha = \frac{r}{p}$$

$$r - r \cos^p \alpha + \cos^p \alpha = \frac{r}{p}$$

$$\cos^p \alpha = \frac{r - \frac{r}{p}}{r} = \frac{1}{p}$$

$$\sin^p \alpha = 1 - \frac{1}{p} = \frac{p-1}{p}$$

$$\tan^p \alpha = \frac{\frac{p-1}{p}}{\frac{1}{p}} = \frac{p-1}{1}$$

$$\frac{\sin^r \alpha + r \cos^r \alpha}{1 + \cos^r \alpha}$$

$$\frac{\cos^r \alpha + r \sin^r \alpha}{1 + \sin^r \alpha} =$$

(9)

$$\frac{\sin^r \alpha + r(1 - \sin^r \alpha)}{1 + \cos^r \alpha}$$

$$\frac{\cos^r \alpha + r(1 - \cos^r \alpha)}{1 + \sin^r \alpha} =$$

$$\frac{\sin^r \alpha + r - r \sin^r \alpha}{1 + \cos^r \alpha}$$

$$\frac{\cos^r \alpha + r - r \cos^r \alpha}{1 + \sin^r \alpha} =$$

$$\frac{(r - \sin^r \alpha)^r}{1 + \cos^r \alpha - \sin^r \alpha}$$

$$\frac{(r - \cos^r \alpha)^r}{1 + \sin^r \alpha - \cos^r \alpha} =$$

$$\frac{(r - \sin^r \alpha)^r}{r - \sin^r \alpha}$$

$$\frac{(r - \cos^r \alpha)^r}{r - \cos^r \alpha} = r - \sin^r \alpha - (r - \cos^r \alpha) =$$

$$r - \sin^r \alpha - r + \cos^r \alpha =$$

$$\cos^r \alpha - \sin^r \alpha = \cos^r \alpha$$

$$\tan \alpha = \frac{r}{p}, \quad \sec \alpha$$

(10)

$$1 + \tan^2 \alpha = \sec^2 \alpha \rightarrow 1 + \frac{r^2}{p^2} = \frac{r^2}{p^2} \rightarrow \cos^2 \alpha = \frac{p^2}{r^2}$$

$$\sin\left(\frac{9\pi}{r} + \alpha\right) \cos\left(\frac{9\pi}{r} - \alpha\right) - \tan\left(\alpha - \frac{9\pi}{r}\right) =$$

$$\cos \alpha \times \sin \alpha + \cot \alpha =$$

$$\begin{cases} \cos \alpha = \frac{-r}{a} \\ \sin \alpha = \frac{-r}{a} \end{cases}$$

$$\downarrow \frac{-r}{a} \times \frac{r}{a} + \frac{r}{r} =$$

$$\frac{-r^2}{a^2} + \frac{r}{r} = \frac{-r^2 + r^2}{a^2} = \frac{0}{a^2} = 0$$

$$\sqrt{r} \cos \alpha + \sqrt{r} \sin \alpha - \sqrt{r} \cos \alpha$$

$$\alpha = \frac{\pi}{4}$$

(1)

$$\sqrt{r} \cos \frac{\pi}{4} + \sqrt{r} (\sin \alpha - \cos \alpha)$$

$$\sqrt{r} \times \frac{1}{\sqrt{2}} + \sqrt{r} (\sqrt{r} \sin(\alpha - \frac{\pi}{4})) =$$

$$\frac{\sqrt{r}}{\sqrt{2}} + \sqrt{r} (\sqrt{r} \sin(\frac{\pi - \pi}{4})) = \frac{\sqrt{r}}{\sqrt{2}} + \sqrt{r} (\sqrt{r} \sin(-\frac{\pi}{4})) =$$

$$\frac{\sqrt{r}}{\sqrt{2}} + \sqrt{r} \times \frac{-\sqrt{r}}{\sqrt{2}} = \frac{\sqrt{r}}{\sqrt{2}} - 1 = \boxed{\frac{1}{\sqrt{2}}}$$

$$\tan\left(\frac{\alpha}{4}\right) = \frac{1}{\sqrt{2}}$$

$$\frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha}$$

$$\frac{1}{10} - \frac{1}{14} = \frac{14 \times 1 - 1 \times 10}{10 \times 14}$$

$$\frac{1}{14} - \frac{10}{14} = \frac{-9}{14}$$

$$= \frac{1 \times 1}{10 \times 14} = \frac{14}{10 \times 14} = \boxed{\frac{-14}{10 \times 14}}$$

$$\tan\left(\frac{\alpha}{4}\right) = \frac{1 - \cos \alpha}{1 + \cos \alpha} = \frac{1}{14} \quad (2)$$

$$14 - 14 \cos \alpha = 1 + \cos \alpha$$

$$14 = 15 \cos \alpha$$

$$\frac{14}{15} = \cos \alpha$$

$$\cos^2 \alpha + \sin^2 \alpha = 1$$

$$\sin^2 \alpha = 1 - \frac{196}{225} = \frac{29}{225}$$

$$\sin \alpha = \frac{1}{15}$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{1}{15}}{\frac{14}{15}} = \frac{1}{14}$$

$$r \sin \alpha < \sin \alpha$$

$$r \sin \alpha < r \sin \alpha \cos \alpha$$

(10)

$$\alpha < \frac{\cot \alpha}{\sin \alpha}$$

$$r \sin \alpha (1 - \cos \alpha) < 0$$

$$\underbrace{\cos \alpha}_{+}$$

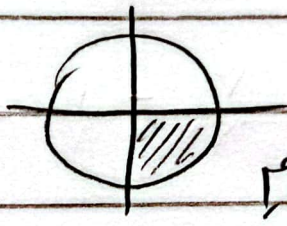
$$\alpha < \frac{\cos \alpha}{\sin^2 \alpha} +$$

$$\cos \alpha > 0$$

$$\rightarrow r \sin \alpha < 0$$

$$\rightarrow \sin \alpha < 0$$

$$\left. \begin{array}{l} \cos \alpha > 0 \\ \sin \alpha < 0 \end{array} \right\}$$



Legend