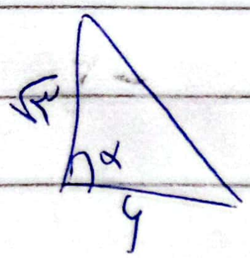
کیارش معصومی — یازدهم دفتر A — ۲۰ — تکالیف شماره ۲۷

(۱)



$$\frac{1}{2}\times 6\times\sqrt{3}\times\sin\alpha=\frac{45}{10}=\frac{9}{2}$$

$$\sin\alpha=\frac{9}{2\times3\sqrt{3}}$$

$$\sin\alpha=\frac{\sqrt{3}}{2}$$

د

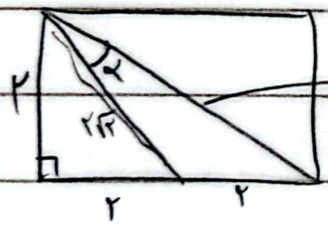
$\sin\alpha=\frac{\sqrt{3}}{2}\rightarrow$ کمترین $\alpha=\frac{\pi}{3}$

$\sin\alpha=\frac{\sqrt{3}}{2}\rightarrow$ بیشترین $\alpha=\frac{2\pi}{3}$

$$\frac{\text{بیشترین } \alpha}{\text{کمترین } \alpha}=\frac{\frac{2\pi}{3}}{\frac{\pi}{3}}=\boxed{2}$$

(۲)

$\cot\alpha=?$



$$\sqrt{2^2+4^2}=2\sqrt{5}$$

د

$$c=\sqrt{a^2+b^2-2ab\cos\alpha}\longrightarrow 4=(2\sqrt{2})^2+(2\sqrt{5})^2-2\times2\sqrt{2}\times2\sqrt{5}\times\cos\alpha$$

$$4=8+20-8\sqrt{10}\times\cos\alpha$$

$$-24=-8\sqrt{10}\times\cos\alpha$$

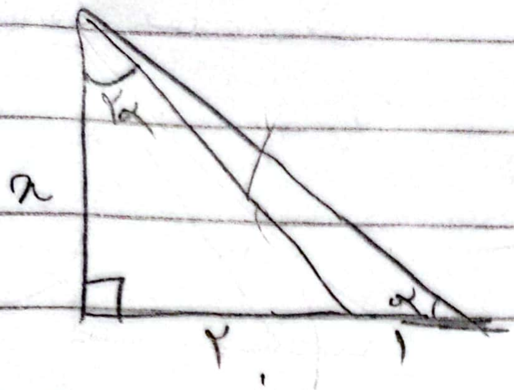
$$\frac{24}{8\sqrt{10}}=\cos\alpha\rightarrow\cos\alpha=\frac{3}{\sqrt{10}}$$

$$\cos^2\alpha+\sin^2\alpha=1\longrightarrow\sin^2\alpha=1-\frac{9}{10}=\frac{1}{10}$$

$$\sin\alpha=\frac{1}{\sqrt{10}}$$

$$\cot\alpha=\frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}}=\boxed{3}$$

۳) $\cot\alpha$ ؟



$$\cot\alpha - \tan\alpha = ۲\cot ۲\alpha$$

۲

$$\frac{۳}{x} - \frac{x}{۳} = ۲\times\frac{x}{۲}$$

$$\frac{۹-x^{۲}}{۳x} = x \rightarrow ۹-x^{۲} = ۳x^{۲}$$

$$۹ = ۴x^{۲}$$

$$\frac{۹}{۴} = x^{۲} \rightarrow x = \frac{۳}{۲}$$

$$\cot\alpha = \frac{۳}{\frac{۳}{۲}} = \frac{۶}{۳} = \boxed{۲}$$

موضوع:

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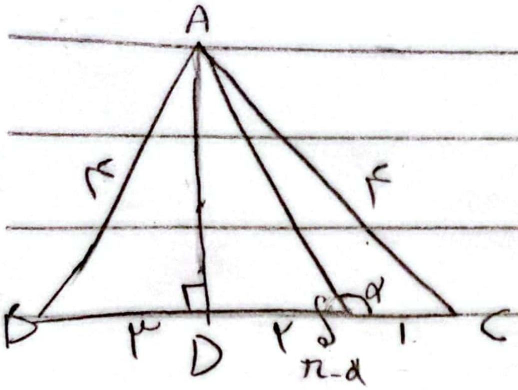

۲

$$AD = \sqrt{۴^۲ - ۳^۲} = \sqrt{۱۶ - ۹} = \sqrt{۷}$$

$$\tan(\pi - \alpha) = -\tan\alpha = \boxed{-\frac{\sqrt{۷}}{۳}}$$

۵

$$۲\sin^۲ x + \cos^۲ x = \frac{۴}{۳}$$

$$۲(۱ - \cos^۲ x) + \cos^۲ x = \frac{۴}{۳}$$

$$۲ - ۲\cos^۲ x + \cos^۲ x = \frac{۴}{۳}$$

$$\cos^۲ x = ۲ - \frac{۴}{۳} = \frac{۲}{۳}$$

$$\sin^۲ x = ۱ - \frac{۲}{۳} = \frac{۱}{۳}$$

$$\tan^۲ x = \frac{\frac{۱}{۳}}{\frac{۲}{۳}} = \boxed{\frac{۱}{۲}}$$

۲

(6)

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} =$$

$$\frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+\sin^2\alpha} =$$

$$\frac{\sin^4\alpha + 4 - 4\sin^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4 - 4\cos^2\alpha}{1+\sin^2\alpha} =$$

$$\frac{(2-\sin^2\alpha)^2}{1+\underbrace{\cos^2\alpha}_{1-\sin^2\alpha}} - \frac{(2-\cos^2\alpha)^2}{1+\underbrace{\sin^2\alpha}_{1-\cos^2\alpha}} =$$

$$\frac{(2-\sin^2\alpha)^2}{2-\sin^2\alpha} - \frac{(2-\cos^2\alpha)^2}{2-\cos^2\alpha} = 2-\sin^2\alpha - (2-\cos^2\alpha) =$$

$$2-\sin^2\alpha - 2 + \cos^2\alpha =$$

$$\cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

(7)

$\tan\alpha = \frac{4}{3}$ ، ربع سوم α

$$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \to 1+\frac{16}{9} = \frac{25}{9} \to \cos^2\alpha = \frac{9}{25}$$

$$\sin\left(\frac{9\pi}{2}+\alpha\right)\cos\left(\frac{7\pi}{2}-\alpha\right) - \tan\left(\alpha - \frac{3\pi}{2}\right) =$$

$$\cos\alpha \times -\sin\alpha + \cot\alpha =$$

$$\begin{cases}\cos\alpha = \frac{-3}{5}\\ \sin\alpha = \frac{-4}{5}\end{cases}$$

$$\frac{-3}{5} \times \frac{4}{5} + \frac{3}{4} =$$

$$\frac{-12}{25} + \frac{3}{4} = \frac{-48+75}{100}$$

$$= \boxed{\frac{27}{100}}$$

$3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x \qquad x = \frac{\pi}{12}$ ①

$3\cos\frac{\pi}{3} + \sqrt{2}(\sin x - \cos x)$

$3 \times \frac{1}{2} + \sqrt{2}\left(\sqrt{2}\sin\left(\alpha - \frac{\pi}{4}\right)\right) =$

$\frac{3}{2} + \sqrt{2}\left(\sqrt{2}\sin\left(\frac{\pi - 3\pi}{12}\right)\right) = \frac{3}{2} + \sqrt{2}\left(\sqrt{2}\sin\left(\frac{-\pi}{6}\right)\right) =$

$\frac{3}{2} + \sqrt{2} \times \frac{-\sqrt{2}}{2} = \frac{3}{2} - 1 = \boxed{\frac{1}{2}}$

$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{4}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} =$

$= \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{17 \times 8 - 8 \times 15}{15 \times 17}}{\frac{-7}{17}}$

$= \frac{\frac{2 \times 8}{15 \times \cancel{17}}}{\frac{-7}{\cancel{17}}} = -\frac{16}{15 \times 7} = \boxed{-\frac{16}{105}}$

$\tan^2\left(\frac{\alpha}{2}\right) = \frac{1 - \cos\alpha}{1 + \cos\alpha} = \frac{1}{16}$ ②

$16 - 16\cos\alpha = 1 + \cos\alpha$

$15 = 17\cos\alpha$

$\frac{15}{17} = \cos\alpha$

$\cos^2\alpha + \sin^2\alpha = 1$

$\sin^2\alpha = 1 - \frac{225}{289} = \frac{64}{289}$

$\sin\alpha = \frac{8}{17}$

$\tan\alpha = \frac{\sin\alpha}{\cos\alpha} = \frac{\frac{8}{17}}{\frac{15}{17}} = \frac{8}{15}$

۱۵

$2\sin\alpha < \sin 2\alpha \rightarrow 2\sin\alpha < 2\sin\alpha\cos\alpha$

$0 < \frac{\cot\alpha}{\sin\alpha}$

$2\sin\alpha(\underbrace{1-\cos\alpha}_{+}) < 0$

$\cos\alpha > 0$

$\downarrow$

$0 < \frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{+}} \Rightarrow \cos\alpha > 0 \rightarrow 2\sin\alpha < 0 \rightarrow \sin\alpha < 0$

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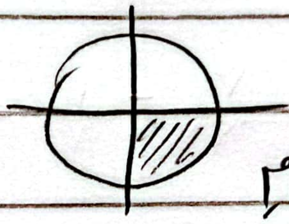
$\begin{cases} \cos\alpha > 0 \\ \sin\alpha < 0 \end{cases}$



ربع چهارم