شهناز محمدرضایی — گروه یازدهم دختر A — تکلیف ۲۷

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۱)

$$S=\frac{۱}{۲}\times AB\times BC\times \sin\alpha=\frac{۱}{\cancel{۲}}\times\sqrt{۳}\times\cancel{۶}^{۳}\times\sin\alpha=\cancel{۴/۵}^{۱/۵}$$

$$\longrightarrow \sin\alpha=\frac{۱/۵}{\sqrt{۳}}=\frac{\sqrt{۳}}{۲}\longrightarrow \alpha=۶۰^\circ \text{ یا } \alpha=۱۲۰^\circ \qquad \boxed{\frac{۱۲۰}{۶۰}=۲}$$

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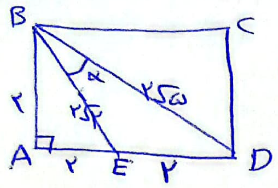
۲)



درمثلث $\overset{\triangle}{ABE} \Longrightarrow BE^۲=AB^۲+AE^۲=۴+۴=۸\longrightarrow BE=۲\sqrt{۲}$

درمثلث $\overset{\triangle}{BCD} \Longrightarrow BD^۲=CD^۲+BC^۲=۱۶+۴=۲۰\longrightarrow BD=۲\sqrt{۵}$

قضیه cos ها درمثلث $E\hat{A}C$ $\longrightarrow ED^۲=BE^۲+BD^۲-۲BE\times BD\times\cos\alpha\longrightarrow ۴=۸+۲۰-۲\times۲\sqrt{۲}\cdot\times۲\sqrt{۵}\times\cos\alpha$

$$\longrightarrow ۲۴=۸\sqrt{۱۰}\cos\alpha\longrightarrow\cos\alpha=\frac{۳}{\sqrt{۱۰}}$$

$$\sin^۲\alpha=۱-\cos^۲\alpha\longrightarrow\sin^۲\alpha=۱-\frac{۹}{۱۰}\longrightarrow\sin\alpha=\frac{۱}{\sqrt{۱۰}}$$

چون α در ناحیه‌ی اول است پس $\sin\alpha>۰$.

$$\cot\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{\frac{۳}{\sqrt{۱۰}}}{\frac{۱}{\sqrt{۱۰}}}=\boxed{۳}$$

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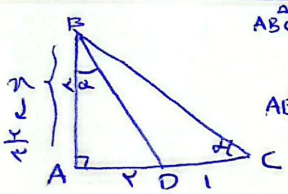
۳)



درمثلث $\overset{\triangle}{ABC}\longrightarrow\cot\alpha=\frac{AC}{AB}=\frac{۳}{x}\longrightarrow\tan\alpha=\frac{۱}{\cot\alpha}=\frac{x}{۳}$

درمثلث $\overset{\triangle}{ABD}\longrightarrow\cot۲\alpha=\frac{AB}{AD}=\frac{x}{۲}\longrightarrow\tan۲\alpha=\frac{۱}{\cot۲\alpha}=\frac{۲}{x}$

$$\frac{۲}{x}=\tan۲\alpha=\frac{۲\tan\alpha}{۱-\tan^۲\alpha}=\frac{۲(\frac{x}{۳})}{۱-(\frac{x}{۳})^۲}=\frac{\frac{۲x}{۳}}{\frac{۹-x^۲}{۹}}\longrightarrow\frac{۶x}{۹-x^۲}=\frac{\cancel{۲}}{x}\longrightarrow ۳x^۲=۹-x^۲\longrightarrow ۴x^۲=۹\longrightarrow x^۲=\frac{۹}{۴}$$

$$x=\pm\frac{۳}{۲}\qquad \cot\alpha=\frac{۳}{x}=\frac{۳}{\frac{۳}{۲}}=\boxed{۲}$$

چون طول ضلع باید مثبت باشد

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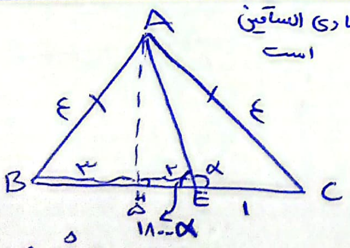
۴)



چون $\overset{\triangle}{ABC}$ متساوی الساقین است $\Longrightarrow AH$ ارتفاع و عمود منصف BC است $\longrightarrow BH=CH=\frac{BC}{۲}$

$BC=۵+۱=۶$ $\Big\}\ BH=CH=۳$

$$\cancel{BE}^{۵}=\cancel{BH}^{۳}+EH\longrightarrow EH=۲$$

درمثلث $A\hat{H}B\longrightarrow AB^۲=BH^۲+AH^۲\longrightarrow ۱۶=۹+AH^۲\longrightarrow AH=\sqrt{۷}$

$$\tan(۱۸۰-\alpha)=\frac{AH}{HD}=\frac{\sqrt{۷}}{۲}\longrightarrow\tan(۱۸۰-\alpha)=-\tan\alpha\longrightarrow-\tan\alpha=\frac{\sqrt{۷}}{۲}$$

$$\boxed{\tan\alpha=-\frac{\sqrt{۷}}{۲}}$$

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۵)

$$۲\sin^۲\alpha+\underbrace{\cos^۲\alpha}_{۱-\sin^۲}=\frac{۴}{۳}\longrightarrow ۲\sin^۲\alpha+۱-\sin^۲\alpha=\frac{۴}{۳}\longrightarrow\sin^۲\alpha=\frac{۱}{۳}$$

$$\cos^۲\alpha=۱-\underbrace{\sin^۲\alpha}_{\frac{۱}{۳}}\longrightarrow\cos^۲\alpha=\frac{۲}{۳}$$

$$\tan^۲\alpha=\frac{\sin^۲\alpha}{\cos^۲\alpha}=\frac{\frac{۱}{۳}}{\frac{۲}{۳}}=\boxed{\frac{۱}{۲}}$$

فرناز محمد رضایی

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$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} - \frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = ?$$

$$\frac{\sin^4\alpha + 4\cos^2\alpha}{1+\cos^2\alpha} = \frac{(\sin^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} = \frac{(1-\cos^2\alpha)^2 + 4\cos^2\alpha}{1+\cos^2\alpha} = \frac{\cos^4\alpha + 1 - 2\cos^2\alpha + 4\cos^2\alpha}{1+\cos^2\alpha}$$

$$= \frac{\cos^4\alpha + 1 + 2\cos^2\alpha}{1+\cos^2\alpha} = \frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha} = \boxed{1+\cos^2\alpha}$$

$$\frac{\cos^4\alpha + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{(\cos^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{(1-\sin^2\alpha)^2 + 4\sin^2\alpha}{1+\sin^2\alpha} = \frac{\sin^4\alpha + 1 - 2\sin^2\alpha + 4\sin^2\alpha}{1+\sin^2\alpha}$$

$$\frac{\sin^4\alpha + 1 + 2\sin^2\alpha}{1+\sin^2\alpha} = \frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha} = 1+\sin^2\alpha$$

ساده شده عبارت $\longrightarrow$ $1+\cos^2\alpha - (1+\sin^2\alpha) = \cos^2\alpha - \sin^2\alpha = \boxed{\cos 2\alpha}$

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$$\underbrace{\sin\left(\frac{9\pi}{2}+\alpha\right)}_{\cos\alpha} \times \underbrace{\cos\left(\frac{7\pi}{2}-\alpha\right)}_{-\sin\alpha} - \underbrace{\tan\left(\alpha - \frac{3\pi}{2}\right)}_{-\cot\alpha} = -\cos\alpha\sin\alpha + \cot\alpha$$

$$\tan\alpha = \frac{4}{3} \longrightarrow \cot\alpha = \frac{1}{\tan\alpha} = \frac{1}{\frac{4}{3}} = \frac{3}{4}$$

در ناحیه ۳ sin و cos منفی

$$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \longrightarrow 1+\frac{16}{9} = \frac{1}{\cos^2\alpha} \longrightarrow \cos\alpha = -\frac{3}{5}$$

$$1+\cot^2\alpha = \frac{1}{\sin^2\alpha} \longrightarrow 1+\frac{9}{16} = \frac{1}{\sin^2\alpha} \longrightarrow \sin\alpha = -\frac{4}{5}$$

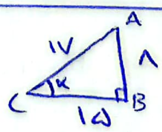
$$\longrightarrow -\cos\alpha\sin\alpha + \cot\alpha = -\left(\frac{-3}{5} \times \frac{-4}{5}\right) + \frac{3}{4} = -\frac{12}{25} + \frac{3}{4} = \frac{-48+75}{100} = \boxed{\frac{27}{100}}$$

(۸

$$\underbrace{3\cos\frac{4\times\pi}{12}}_{\underbrace{3\cos\frac{\pi}{3}}_{\frac{3}{2}}} + \underbrace{\sqrt{2}\sin\frac{\pi}{12} - \sqrt{2}\cos\frac{\pi}{12}}_{\sqrt{2}\underbrace{\left(\sin\frac{\pi}{12} - \cos\frac{\pi}{12}\right)}_{\sqrt{2}\sin\underbrace{\left(\frac{\pi}{12}-\frac{\pi}{4}\right)}_{-\frac{\pi}{6}}}} = \frac{3}{2} + \sqrt{2}\left(\sqrt{2} \times \frac{-1}{2}\right) = \frac{3}{2} - 1 = \frac{1}{2}$$

(۹

$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{2\times\frac{1}{4}}{1-\frac{1}{16}} = \frac{\frac{1}{2}}{\frac{15}{16}} = \frac{8}{15}$$



$$AC^2 = \underbrace{AB^2}_{225} + \underbrace{BC^2}_{64} \longrightarrow AC = 17$$

$$\sin\alpha = \frac{8}{17} \qquad \cos\alpha = \frac{15}{17}$$

چون $0 < \tan\frac{\alpha}{2} < 1 \Leftarrow 180^\circ < \frac{\alpha}{2} < 270^\circ \Leftarrow 360^\circ < \alpha < 540^\circ \Leftarrow$ ناحیه اول sin و cos مثبت

$$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{8\times17 - 15\times8}{\cancel{17}\times15}}{\frac{-7}{\cancel{17}}} = \boxed{\frac{-16}{105}}$$

۱۰)

$\dfrac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \dfrac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \rightarrow \dfrac{\cos\alpha}{\underbrace{\sin^2\alpha}_{+}} > 0 \rightarrow \cos\alpha > 0 \rightarrow$ ناحیه ۱ یا ۴ ①

$2\sin\alpha < \sin 2\alpha \rightarrow 2\sin\alpha - \sin 2\alpha < 0 \rightarrow 2\sin\alpha - 2\sin\cos\alpha < 0$

$\rightarrow 2\sin\alpha(\underbrace{1-\cos\alpha}_{+}) < 0 \rightarrow \sin\alpha < 0 \rightarrow$ ناحیه ۳ یا ۴ ②

$① \cap ②$

$\downarrow$

ناحیه ۴