مهناز محمدرضایی — گروه یازدهم دختر A — تکلیف ۲۷

۲۰

۱)

$S=\frac{1}{2}\times AB\times BC\times\sin\alpha=\frac{1}{\cancel{2}}\times\sqrt{3}\times\cancel{6}^{3}\times\sin\alpha=\cancel{4/5}^{1/5}$

$\longrightarrow \sin\alpha=\frac{1/5}{\sqrt{3}}=\frac{\sqrt{3}}{2}\longrightarrow \alpha=60^\circ$ یا $\alpha=120^\circ$ $\frac{120}{60}=2$

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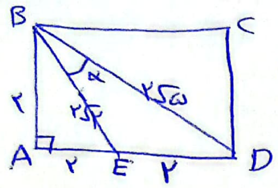
۲)



در مثلث $A\hat{B}E \Longrightarrow BE^2=AB^2+AE^2=4+4=8\longrightarrow BE=2\sqrt{2}$

در مثلث $B\hat{C}D \Longrightarrow BD^2=CD^2+BC^2=16+4=20\longrightarrow BD=2\sqrt{5}$

قضیه cos ها در مثلث $E\hat{A}C$ $\longrightarrow ED^2=BE^2+BD^2-2BE\times BD\times\cos\alpha\longrightarrow 4=8+20-2\times2\sqrt{2}\cdot\times2\sqrt{5}\times\cos\alpha$

$\longrightarrow 24=8\sqrt{10}\cos\alpha\longrightarrow\cos\alpha=\frac{3}{\sqrt{10}}$ $\sin^2\alpha=1-\cos^2\alpha\longrightarrow\sin^2\alpha=1-\frac{9}{10}\longrightarrow\sin\alpha=\frac{1}{\sqrt{10}}$

چون α در ناحیه‌ی اول است پس $\sin\alpha>0$.

$\cot\alpha=\frac{\cos\alpha}{\sin\alpha}=\frac{\frac{3}{\sqrt{10}}}{\frac{1}{\sqrt{10}}}=3$

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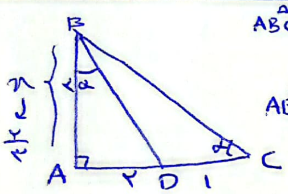
۳)



در مثلث $A\hat{B}C\rightarrow\cot\alpha=\frac{AC}{AB}=\frac{3}{x}\longrightarrow\tan\alpha=\frac{1}{\cot\alpha}=\frac{x}{3}$

در مثلث $A\hat{B}D\rightarrow\cot2\alpha=\frac{AB}{AD}=\frac{x}{2}\longrightarrow\tan2\alpha=\frac{1}{\cot2\alpha}=\frac{2}{x}$

$\frac{2}{x}=\tan2\alpha=\frac{2\tan\alpha}{1-\tan^2\alpha}=\frac{2(\frac{x}{3})}{1-(\frac{x}{3})^2}=\frac{\frac{2x}{3}}{\frac{9-x^2}{\cancel{9}^{3}}}\longrightarrow\frac{6x}{9-x^2}=\frac{\cancel{2}}{x}\longrightarrow 3x^2=9-x^2\longrightarrow 4x^2=9\rightarrow x^2=\frac{9}{4}$

$x=\pm\frac{3}{2}$ $\cot\alpha=\frac{3}{x}=\frac{3}{\frac{3}{2}}=2$

چون طول ضلع باید مثبت باشد

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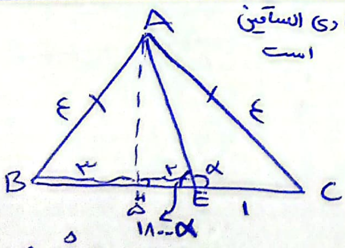
۴)



چون $A\hat{B}C$ متساوی الساقین است $\Longrightarrow AH$ ارتفاع و عمود منصف BC است $\longrightarrow BH=CH=\frac{BC}{2}$ ، $BC=5+1=6$ $\Big\}$ $BH=CH=3$

$\cancel{BE}^{5}=\cancel{BH}^{3}+EH\longrightarrow EH=2$

در مثلث $A\hat{H}B\longrightarrow AB^2=BH^2+AH^2\rightarrow 16=9+AH^2\longrightarrow AH=\sqrt{7}$

$\tan(180-\alpha)=\frac{AH}{HE}=\frac{\sqrt{7}}{2}\longrightarrow\tan(180-\alpha)=-\tan\alpha\longrightarrow-\tan\alpha=\frac{\sqrt{7}}{2}$

$\tan\alpha=-\frac{\sqrt{7}}{2}$

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۵)

$2\sin^2\alpha+\underbrace{\cos^2\alpha}_{1-\sin^2}=\frac{4}{3}\longrightarrow 2\sin^2\alpha+1-\sin^2\alpha=\frac{4}{3}\longrightarrow\sin^2\alpha=\frac{1}{3}$

$\cos^2\alpha=1-\underbrace{\sin^2\alpha}_{\frac{1}{3}}\longrightarrow\cos^2\alpha=\frac{2}{3}$

$\tan^2\alpha=\frac{\sin^2\alpha}{\cos^2\alpha}=\frac{\frac{1}{3}}{\frac{2}{3}}=\frac{1}{2}$

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6)

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}-\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=?$$

$$\frac{\sin^4\alpha+4\cos^2\alpha}{1+\cos^2\alpha}=\frac{(\sin^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}=\frac{(1-\cos^2\alpha)^2+4\cos^2\alpha}{1+\cos^2\alpha}=\frac{\cos^4\alpha+1-2\cos^2\alpha+4\cos^2\alpha}{1+\cos^2\alpha}$$

$$=\frac{\cos^4\alpha+1+2\cos^2\alpha}{1+\cos^2\alpha}=\frac{(1+\cos^2\alpha)^2}{1+\cos^2\alpha}=\boxed{1+\cos^2\alpha}$$

2

$$\frac{\cos^4\alpha+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{(\cos^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{(1-\sin^2\alpha)^2+4\sin^2\alpha}{1+\sin^2\alpha}=\frac{\sin^4\alpha+1-2\sin^2\alpha+4\sin^2\alpha}{1+\sin^2\alpha}$$

$$\frac{\sin^4\alpha+1+2\sin^2\alpha}{1+\sin^2\alpha}=\frac{(1+\sin^2\alpha)^2}{1+\sin^2\alpha}=1+\sin^2\alpha$$

ساده شده عبارت $\longrightarrow$ $1+\cos^2\alpha-(1+\sin^2\alpha)=\cos^2\alpha-\sin^2\alpha=\boxed{\cos 2\alpha}$

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7)

$$\underbrace{\sin\left(\frac{9\pi}{2}+\alpha\right)}_{\cos\alpha}\times\underbrace{\cos\left(\frac{7\pi}{2}-\alpha\right)}_{-\sin\alpha}-\underbrace{\tan\left(\alpha-\frac{3\pi}{2}\right)}_{-\cot\alpha}=-\cos\alpha\sin\alpha+\cot\alpha$$

1

در ناحیه ۳ sin و cos منفی

$$\tan\alpha=\frac{4}{3}\longrightarrow\cot\alpha=\frac{1}{\tan\alpha}=\frac{1}{\frac{4}{3}}=\frac{3}{4}$$

$$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\longrightarrow 1+\frac{16}{9}=\frac{1}{\cos^2\alpha}\longrightarrow\cos\alpha=-\frac{3}{5}$$

$$1+\cot^2\alpha=\frac{1}{\sin^2\alpha}\longrightarrow 1+\frac{9}{16}=\frac{1}{\sin^2\alpha}\longrightarrow\sin\alpha=-\frac{4}{5}$$

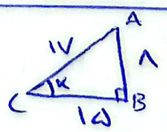
$$\longrightarrow -\cos\alpha\sin\alpha+\cot\alpha=-\left(\frac{-3}{5}\times\frac{-4}{5}\right)+\frac{3}{4}=-\frac{12}{25}+\frac{3}{4}=\frac{-48+75}{100}=\boxed{\frac{27}{100}}$$

$$\underbrace{3\cos\left(4\times\frac{\pi}{12}\right)}_{3\cos\frac{\pi}{3}=\frac{3}{2}}+\underbrace{\sqrt{2}\sin\frac{\pi}{12}-\sqrt{2}\cos\frac{\pi}{12}}_{\sqrt{2}\,\underbrace{\left(\sin\frac{\pi}{12}-\cos\frac{\pi}{12}\right)}_{\sqrt{2}\sin\left(\frac{\pi}{12}-\frac{\pi}{4}\right)}}=\frac{3}{2}+\sqrt{2}\left(\sqrt{2}\times\frac{-1}{2}\right)=\frac{3}{2}-1=\frac{1}{2}$$

$$\underbrace{\frac{\pi}{12}-\frac{\pi}{4}}_{-\frac{\pi}{6}}$$

2

$$\tan\alpha=\frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}}=\frac{2\times\frac{1}{4}}{1-\frac{1}{16}}=\frac{\frac{1}{2}}{\frac{15}{16}}=\frac{8}{15}$$



$$AC^2=\underbrace{AB^2}_{225}+\underbrace{BC^2}_{64}\longrightarrow AC=17$$

$$\sin\alpha=\frac{8}{17}\qquad\cos\alpha=\frac{15}{17}$$

2

چون $0<\tan\frac{\alpha}{2}<1 \Leftarrow 180<\frac{\alpha}{2}<270 \Leftarrow 360<\alpha<540 \Leftarrow$ ناحیه اول sin و cos مثبت

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}}=\frac{\frac{8\times17-15\times8}{\cancel{17}\times15}}{\frac{-7}{\cancel{17}}}=\boxed{\frac{-16}{105}}$$

۱۰)

$$\frac{\cot\alpha}{\sin\alpha} > 0 \Rightarrow \frac{\frac{\cos\alpha}{\sin\alpha}}{\sin\alpha} > 0 \rightarrow \frac{\cos\alpha}{\underbrace{\sin^2\alpha}_{+}} > 0 \rightarrow \cos\alpha > 0 \rightarrow$$ ناحیه ۱ یا ۴ ①

$$2\sin\alpha < \sin 2\alpha \rightarrow 2\sin\alpha - \sin 2\alpha < 0 \rightarrow 2\sin\alpha - 2\sin\cos\alpha < 0$$

$$\rightarrow 2\sin\alpha\,(\underbrace{1-\cos\alpha}_{+}) < 0 \rightarrow \sin\alpha < 0 \rightarrow$$ ناحیه ۳ یا ۴ ②

د

$$① \cap ② \downarrow$$ ناحیه ۴