گروه یازدهم دهم A

شماره کلاسی: ۲۷

نام: امیرحسین

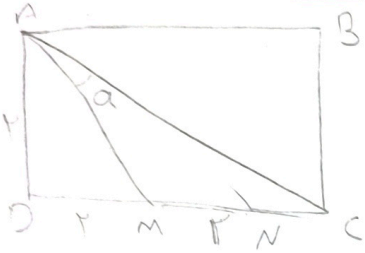
۲۰

(۱)

$S = \frac{1}{2}ab\sin\alpha \Rightarrow S = \frac{1}{2}\times 4\times\sqrt{12}\times\sin\alpha = \frac{9}{2} \Rightarrow \sin\alpha = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$

$\min\sin\alpha = 60^\circ$, $\max\sin\alpha = 120^\circ \longrightarrow \frac{120}{60} = 2$

(۲)



$AN^2 = 4 + 16 = 20 \rightarrow AN = \sqrt{20}$

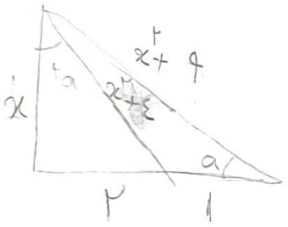
$AM^2 = 16 + 4 - 8 \Rightarrow AM = \sqrt{8}$

قانون کسینوس‌ها $\rightarrow 8 = 20 + 8 - 2\sqrt{20}\times\sqrt{8}\times\cos\alpha \Rightarrow 4 = 28 - 4\sqrt{40}\cos\alpha$

$12 = \sqrt{40}\cos\alpha \rightarrow 12 = 6\sqrt{1-\cos^2\alpha} \Rightarrow \frac{3}{\sqrt{10}} = \cos\alpha$

$\cos^2\alpha = \frac{9}{10}$ $\sin^2\alpha = \frac{1}{10} \Rightarrow \cot^2\alpha = \frac{\cos^2\alpha}{\sin^2\alpha} = \frac{\frac{9}{10}}{\frac{1}{10}} \Rightarrow \cot\alpha = \pm 3 \rightarrow$ وقتی ۳+ می‌گیرد که $\alpha$ حاده است

(۳)

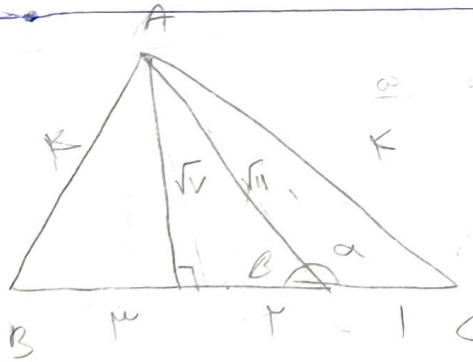


$\cot\alpha = \frac{x}{3}$, $\tan\alpha = \frac{3}{x} \Rightarrow \frac{3}{x} + \frac{x}{3} = 4$

$\frac{9-x^2}{3x} = x \Rightarrow 9 = 4x^2$

$\frac{9}{4} = x^2 \rightarrow \frac{3}{2} = x \longrightarrow \cot\alpha = \frac{3}{\frac{3}{2}} = 2$

(۴) دو زاویه متمم اندازه‌های قطع فرد برعکس با تانژانت‌هایشان برابرند.



$\tan\beta = \frac{\sqrt{7}}{2}$ دو زاویه مکمل cotan برابر و قرینه‌اند

$\tan\alpha = -\frac{\sqrt{7}}{2}$

(۵)

$2\sin^2\alpha + \cos^2\alpha = \frac{4}{3} \rightarrow 2\sin^2\alpha + \sin^2\alpha + \cos^2\alpha = \frac{4}{3} \rightarrow 3\sin^2\alpha = \frac{1}{3}$

$1 - \frac{1}{3} = \frac{2}{3} = \cos^2\alpha$

$\tan^2\alpha = \frac{\frac{1}{3}}{\frac{2}{3}} = \frac{1}{2}$

۴

$$\frac{(1-\cos a)^2+4\cos^2 a}{1+\cos^2 a}-\frac{(1-\sin^2 a)^2+4\sin^2 a}{1+\sin^2 a}$$

$$\frac{1-2\cos^2 a+\cos^4 a+4\cos^2 a}{1+\cos^2 a}-\frac{1-2\sin^2 a+\sin^4 a+4\sin^2 a}{1+\sin^2 a}$$

$$1+\cos^2 a-\sin^2 a-1=\cos^2 a-\sin^2 a=\cos 2a$$

۷

$$\sin\left(\frac{9\pi}{2}+2a\right)\cos\left(\frac{7\pi}{2}-a\right)-\tan\left(a-\frac{3\pi}{2}\right)=\cos a\times(-\sin a)+\cot a$$

$$\cot\alpha=\frac{3}{4}$$

$$1+\tan^2\alpha=\frac{1}{\cos^2\alpha}\rightarrow 1+\frac{16}{9}=\frac{1}{\cos^2\alpha}\rightarrow \cos\alpha=\frac{3}{5}$$

$$1-\frac{9}{25}=\frac{16}{25}\rightarrow \sin\alpha=\frac{4}{5}$$

$$\rightarrow \frac{3}{5}\times\left(-\frac{4}{5}\right)+\frac{3}{4}=\frac{-12}{25}+\frac{3}{4}$$

$$\frac{-48+75}{100}=\frac{27}{100}$$

۸

$$3\cos^2\left(\frac{\pi}{12}\right)+\sqrt{2}\sin\frac{5\pi}{12}-\sqrt{2}\cos\frac{\pi}{12}=\frac{3}{2}+\sqrt{2}(\sin 15-\cos 15)=\frac{3}{2}+\sqrt{2}\sin(15-45)\times\sqrt{2}$$

$$\frac{3}{2}-1=\frac{1}{2}$$

۹

$$1+\left(\tan\frac{\alpha}{2}\right)^2=\frac{1}{\cos^2\frac{\alpha}{2}}\rightarrow 1+\frac{1}{15}=\frac{1}{\cos^2\frac{\alpha}{2}}\rightarrow \cos^2\frac{\alpha}{2}=\frac{15}{16}\rightarrow \sin^2\frac{\alpha}{2}=\frac{1}{16}$$

$$\sin\frac{\alpha}{2}=\frac{\sqrt{15}}{16}$$

$$\sin\alpha=2\times\frac{\sqrt{15}}{16}\times\frac{1}{\sqrt{16}}=\frac{\sqrt{15}}{8}$$

$$\cos\alpha=\sqrt{1-\left(\frac{\sqrt{15}}{16}\right)^2}=\frac{1}{16}$$

$$\frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha}=\frac{\frac{\sqrt{15}}{1}-\frac{\sqrt{15}}{16}}{\frac{\sqrt{15}}{16}-\frac{1}{16}}=\frac{\frac{240-15}{16}}{\frac{\sqrt{15}-1}{16}}=\frac{225}{\sqrt{15}-1}$$

۱۰

$$\cos\alpha>0\leftarrow 0<\frac{\cos\alpha}{\sin^2\alpha}$$

$$2\sin\alpha<\sin^2\alpha$$

$$2\sin\alpha<2\sin^2\alpha$$

$$0<2\sin\alpha(\cos\alpha-1)\leftarrow 2\sin\alpha\cos\alpha-2\sin\alpha<0$$

$$\begin{cases}\sin\alpha<0\\ \cos\alpha>0\end{cases}\leftarrow$$

ربع چهارم