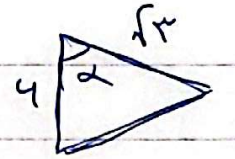
تاریخ: ..... موضوع: .....

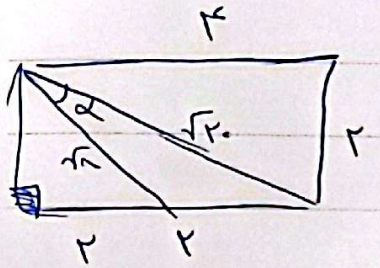


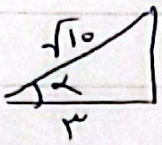
$$S = \frac{1}{2} \times \sqrt{2} \times 4 \times \sin\alpha = \frac{9}{2} \quad (1)$$

$$\sin\alpha = \frac{\sqrt{2}}{2} \begin{cases} \alpha = 45^\circ \\ \alpha = 135^\circ \end{cases} \longrightarrow \frac{135}{45} = 3$$

$$S_{\text{ذوزنقه}} = 4 \times 2 - \left(\frac{2 \times 2}{2} + \frac{4 \times 2}{4} \right) = 8 - 2 - 4 = 2 \quad (2)$$

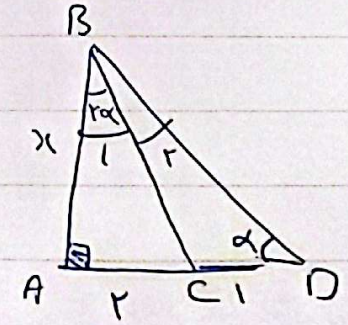
$$S_{\triangle} = \sqrt{20} \times \sqrt{8} \times \frac{1}{2} \times \sin\alpha = 2 \longrightarrow \sin\alpha = \frac{1}{\sqrt{10}}$$





$$\longrightarrow \cot\alpha = \frac{3}{1}$$

(3)



$$\tan B_1 = \frac{AC}{AB} \Rightarrow \tan 2\alpha = \frac{2}{x}$$

$$\tan D = \frac{AB}{AD} \Rightarrow \tan\alpha = \frac{x}{3}$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha} \Rightarrow \frac{2}{x} = \frac{2 \times \frac{x}{3}}{1 - \left(\frac{x}{3}\right)^2} \Rightarrow \frac{2}{x} = \frac{\frac{2x}{3}}{\frac{9 - x^2}{9}} \Rightarrow$$

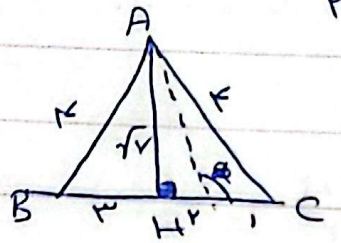
$$\frac{2}{x} = \frac{6x}{9 - x^2} \longrightarrow 18 - 2x^2 = 6x^2 \longrightarrow 8x^2 = 18$$

$$x^2 = \frac{9}{4} \longrightarrow x = \frac{3}{2}$$

$$\cot\alpha = \cot D = \frac{AD}{AB} = \frac{3}{x} \Rightarrow \frac{3}{\frac{3}{2}} \Rightarrow 2$$

۴) چون مثلث متساوی‌الساقین است پس $AB=AC=۴$ پس AH هم میانه است و هم ارتفاع پس:

$$\tan(\pi-\alpha)=-\tan\pi=\frac{\sqrt{۷}}{۳}\Rightarrow\tan\alpha=\frac{-\sqrt{۷}}{۳}$$



۵)

$$۲\sin^۲x+\cos^۲x=\frac{۴}{۳}\xrightarrow{\div\cos^۲\alpha}۲\tan^۲x+۱=\frac{۴}{۳}(۱+\tan^۲x)$$

$$۶\tan^۲x+۳=۴+۴\tan^۲x\longrightarrow ۲\tan^۲x=۱\rightarrow\tan^۲x=\frac{۱}{۲}$$

۶)

$$A=\frac{\sin^۴\alpha+۴\cos^۲\alpha}{۱+\cos^۲\alpha}-\frac{\cos^۴\alpha+۴\sin^۲\alpha}{۱+\sin^۲\alpha}$$

$$A=\frac{(۱-\cos^۲\alpha)+۴\cos^۲\alpha}{۱+\cos^۲\alpha}-\frac{(۱-\sin^۲\alpha)^۲+۴\sin^۲\alpha}{۱+\sin^۲\alpha}$$

$$A=\frac{(۱+\cos^۲\alpha)^۲}{۱+\cos^۲\alpha}-\frac{(۱+\sin^۲\alpha)^۲}{۱+\sin^۲\alpha}=\cos۲\alpha$$

$\cos\alpha = -\frac{\sqrt{5}}{3}$ ← زاویه در ربع دوم

موضوع ..... تاریخ .....

$\tan\alpha = \frac{4}{3}$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \rightarrow 1+\frac{16}{9} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{9}{25}$ (۷)

$\sin^2\alpha = 1-\cos^2\alpha \rightarrow \sin^2\alpha = 1-\frac{9}{25} = \frac{16}{25}$

$\sin\alpha = -\frac{4}{5}$ ← زاویه در ربع سوم

$\cot\alpha = \frac{3}{4}$

$\sin\left(\frac{9\pi}{2}+\alpha\right) = \left(\sin 4\pi + \frac{\pi}{2}+\alpha\right) = \sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha =$

$\Rightarrow -\frac{3}{5}$

$\cos\left(\frac{27\pi}{2}-\alpha\right) = \cos\left(12\pi + \frac{3\pi}{2}-\alpha\right) = \cos\left(\frac{3\pi}{2}-\alpha\right)$

$\Rightarrow -\sin\alpha = \frac{4}{5}$

$\tan\left(\alpha - \frac{7\pi}{2}\right) = -\tan\left(\frac{7\pi}{2}-\alpha\right) = -\cot\alpha = -\frac{3}{4}$

حاصل $= \left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) + \frac{3}{4} = \frac{27}{100} = \%27$

$\cos 8\alpha = \cos\frac{8\pi}{12} = \frac{1}{2}$ (۸)

$\sqrt{2}(\sin\alpha - \cos\alpha) = \sqrt{2}\times\sqrt{2}\sin\left(\alpha - \frac{\pi}{4}\right) = 2\sin\left(-\frac{\pi}{4}\right)$

$= -1$

حاصل $= \frac{3}{2} - 1 = \frac{1}{2}$

۹)

$\tan 2\alpha = \frac{2\tan\alpha}{1-\tan^2\alpha}$, $1+\tan^2\alpha = \frac{1}{\cos^2\alpha}$

$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{\frac{1}{2}}{1-\frac{1}{16}} = \frac{8}{15}$

$1+\tan^2\alpha = \frac{1}{\cos^2\alpha} \Rightarrow 1+\frac{64}{225} = \frac{1}{\cos^2\alpha} \rightarrow \cos^2\alpha = \frac{225}{289}$

$\cos\alpha = \frac{15}{17} \longrightarrow \sin\alpha = \sqrt{1-\frac{225}{289}} = \frac{8}{17}$

$\frac{\tan\alpha - \sin\alpha}{\sin\alpha - \cos\alpha} = \frac{\frac{8}{15} - \frac{8}{17}}{\frac{8}{17} - \frac{15}{17}} = \frac{\frac{16}{15}}{-7} = \frac{-16}{105}$

۱۰) $\sin 2\alpha > 2\sin\alpha$, $\frac{\cot\alpha}{\sin\alpha} > 0$

$\frac{\cot\alpha}{\sin\alpha} = \frac{\cos\alpha}{\sin^2\alpha}$

$2\sin\alpha\cos\alpha > 2\sin\alpha$ ← $\sin 2\alpha = 2\sin\alpha\cos\alpha$ → هر عبارت را یک ۲ ساده کنیم → $\sin\alpha(\cos\alpha - 1) > 0$

$\cos\alpha - 1 < 0$ فقط زمانی صفر می‌شود که cos برابر ۱ باشد.

به دلیل اینکه $\sin^2\alpha$ مثبت است علامت کسر برابر است با علامت $\cos\alpha$

$\cos\alpha > 0$ ← $\frac{\cos\alpha}{\sin\alpha} > 0$ ← $\cos\alpha$

⊕ ←

↓

α در ربع ۴ یا ۱ است

برای اینکه عبارت بزرگتر از صفر باشد باید یکی + و دیگری منفی باشد

$\sin\alpha < 0$

$(\cos\alpha - 1) < 0$

$\cos\alpha > 0$ ← ربع ۱ و ۴

$\sin\alpha < 0$ → ربع سوم یا چهارم

} اشتراک آنها ربع چهارم است!