Date: ۰۴/۱۲/۲۱ Subject: آزاد بنی نژاد

(۱)

$S = \frac{1}{2} \times \sqrt{3} \times 6 \times \sin\alpha = 4/5$

$\frac{120}{60} = \frac{2}{1} \Longleftarrow \quad \sin\alpha = \frac{\sqrt{3}}{2} \rightarrow \alpha = 60$ یا 120

(۲)

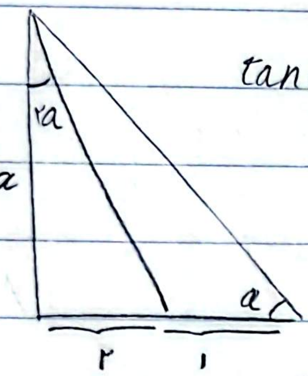
زاویه‌ی سمت چپ α را β گرفتم

$\tan\beta = \frac{2}{2} = 1$

$\tan(\alpha+\beta) = \frac{4}{2} = 2 \quad \tan\alpha = \tan((\alpha+\beta)-\beta) \rightarrow$

$\frac{\tan(\alpha+\beta) - \tan\beta}{1+\tan(\alpha+\beta)\tan\alpha} = \frac{2-1}{1+(2)(1)} = \frac{1}{3} \longrightarrow$ کتانژانت آلفا $= \underline{\underline{3}}$

(۳)



$\tan a = \frac{x}{3} \qquad \tan(2)(a) = \frac{2}{x}$

$\tan 2a = \frac{2\tan a}{1-\tan^2 a} \Rightarrow \frac{2}{x} = \frac{\frac{2x}{3}}{1-\frac{x^2}{9}} \rightarrow$

$\frac{2}{x} = \frac{6x}{9-x^2} \Rightarrow 6x^2 + 2x^2 = 18 \rightarrow x^2 = \frac{9}{4}$

$x = \frac{3}{2} \qquad \tan\alpha = \frac{x}{3} = \frac{1}{2} \qquad \cot\alpha = 2$

(۴)

HA (ارتفاع) را رسم کردم و چپ α را β نام دادم (M نقطه‌ی بین α و β است)

$CH = 6 - 3 = 3 \rightarrow HM = 2 \quad AH^2 + HC^2 = 16 \quad AH^2 + 9 = 16$

$\tan\beta = \frac{AH}{HM} = \frac{\sqrt{7}}{2} \qquad \underbrace{\tan\alpha = \tan(180-\beta)}_{\tan\beta = \frac{-\sqrt{7}}{2}} \leftarrow AH = \sqrt{7}$

(۵)

$\sin^2 x + \underbrace{\sin^2 x + \cos^2 x}_{1} = \frac{4}{3} \longrightarrow \sin^2 x = \frac{1}{3}$

$\cos^2 x = \frac{2}{3}$

$\tan^2 x = \frac{1}{3} \times \frac{3}{2} = \frac{1}{2}$

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$$\frac{\sin^4\alpha + 4(1-\sin^2\alpha)}{1+\cos^2\alpha} = \frac{\cos^4\alpha + 4(1-\cos^2\alpha)}{1+\sin^2\alpha} =$$

$$\frac{(\sin^2\alpha-2)^2}{1+\cos^2\alpha} - \frac{(\cos^2\alpha-2)^2}{1+\sin^2\alpha} = \frac{(-1-\cos^2\alpha)^2}{1+\cos^2\alpha} - \frac{(-1-\sin^2\alpha)^2}{1+\sin^2\alpha}$$

$$= 1+\cos^2\alpha - (+1+\sin^2\alpha) = \cos^2\alpha - \sin^2\alpha = \cos 2\alpha$$

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$\sin\left(\frac{9\pi}{2}+\alpha\right)$ ربع ۲ $\quad \cos\left(\overbrace{\frac{7\pi}{2}}^{\frac{8\pi}{2}-\frac{\pi}{2}}-\alpha\right)$ ربع ۴

$-\tan\left(\alpha-\frac{3\pi}{2}\right) = \tan\left(\frac{3\pi}{2}-\alpha\right)$ ربع ۳

$$-\left(\frac{-3}{5}\right)\left(\frac{4}{5}\right)+\frac{3}{4} = \frac{27}{100} = 0/27$$

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$$\left(3\cos 4x + \sqrt{2}\sin x - \sqrt{2}\cos x\right) \quad x=\frac{\pi}{12}$$

$$3\cos 4x + 2\left(\frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x\right) = 3\cos 4x + 2\sin\left(x-\frac{\pi}{4}\right)$$

$$\to 3\cos\frac{\pi}{3} + 2\sin\left(\frac{-\pi}{6}\right) = 3\left(\frac{1}{2}\right) - 2\left(\frac{1}{2}\right) = \frac{1}{2}$$

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$$\tan\alpha = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\frac{\alpha}{2}} = \frac{\frac{1}{2}}{1-\frac{1}{16}} = \frac{8}{15} \qquad \sin\alpha = \frac{(2)(\tan\frac{\alpha}{2})}{1+\tan^2\frac{\alpha}{2}} = \frac{\frac{1}{2}}{1+\frac{1}{16}}$$

$$= \frac{8}{17} \Rightarrow \cos\alpha = \sqrt{1-\sin^2\alpha} = \frac{15}{17} \longrightarrow \frac{\tan\alpha-\sin\alpha}{\sin\alpha-\cos\alpha} =$$

$$\frac{\frac{8}{15}-\frac{8}{17}}{\frac{8}{17}-\frac{15}{17}} = \frac{-16}{105}$$

۱۰

$$\sin\alpha - \sin\alpha\cos\alpha < 0$$

$\sin\alpha(\underbrace{1-\cos\alpha}_{\text{مثبت}}) < 0 \to \sin\alpha$ منفی

$$\frac{\cot\alpha}{\sin\alpha} > 0 \qquad \cot\alpha < 0 \qquad \frac{\cos\alpha}{\sin\alpha} < 0 \qquad \cos\alpha > 0$$

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