

$$\frac{\sin^2 \alpha + E \cos^2 \alpha}{1 + \cos^2 \alpha} = \frac{\cos^2 \alpha + E \sin^2 \alpha}{\sin^2 \alpha + 1} \Rightarrow \frac{(1 - \cos^2 \alpha)^E + E \cos^2 \alpha}{\cos^2 \alpha + 1} = \frac{(1 - \sin^2 \alpha)^E + E \sin^2 \alpha}{\sin^2 \alpha + 1}$$

$$1 + \frac{\cos^2 \alpha + \cos^2 \alpha + E \cos^2 \alpha}{\cos^2 \alpha + 1} = \frac{(\cos^2 \alpha + 1)^E}{\cos^2 \alpha + 1} = \cos^2 \alpha + 1 \quad \Big/ \quad \frac{1 - \sin^2 \alpha + \sin^2 \alpha + E \sin^2 \alpha}{\sin^2 \alpha + 1} = \frac{(\sin^2 \alpha + 1)^E}{\sin^2 \alpha + 1} = \sin^2 \alpha + 1$$

$$\cos^2 \alpha + \sin^2 \alpha = \cos^2 \alpha - \sin^2 \alpha = \cos 2\alpha$$

(8)

$\tan \alpha = \frac{E}{V} \rightarrow$

$$\sin \alpha = \frac{E}{\sqrt{E^2 + V^2}}, \quad \cos \alpha = \frac{V}{\sqrt{E^2 + V^2}}$$

$$\sin\left(\frac{\pi}{4} + \alpha\right) \times \cos\left(\frac{\pi}{4} - \alpha\right) - \tan\left(\alpha - \frac{\pi}{4}\right) = \cos \alpha \times \sin \alpha + \cot \alpha = -x \frac{V}{E} \times \frac{E}{V} + \frac{V}{E}$$

$$\Rightarrow \frac{-V}{V} + \frac{V}{E} = \frac{-E + V}{E} = \frac{-1 + V}{E}$$

(9)

$$x = \frac{\pi}{12} = \sqrt{\cos\left(\frac{E \times \pi}{V \times \pi}\right)} + \sqrt{\pi} \left(\sin \frac{\pi}{12} - \cos \frac{\pi}{12} \right) = \frac{V}{V} + \sqrt{\pi} \times \left(\frac{1}{2} - \frac{\sqrt{3}}{2} \right) = \frac{V}{V} - 1 = \frac{1}{V}$$

$$\frac{1 - \pi}{12} \pi = \frac{-\pi}{12} = \frac{-\pi}{12}$$

(10)

$\tan\left(\frac{\alpha}{2}\right) = \frac{1}{E} \rightarrow$

$$\left. \begin{aligned} \cos\left(\frac{\alpha}{2}\right) &= \frac{E}{\sqrt{1 + E^2}} \\ \sin\left(\frac{\alpha}{2}\right) &= \frac{1}{\sqrt{1 + E^2}} \end{aligned} \right\} \Rightarrow \sin \alpha = 2 \sin \frac{\alpha}{2} \cos \frac{\alpha}{2} = 2 \times \frac{1}{\sqrt{1 + E^2}} \times \frac{E}{\sqrt{1 + E^2}} = \frac{2E}{1 + E^2}$$

$$\frac{\tan \alpha - \sin \alpha}{\sin \alpha - \cos \alpha} = ? \Rightarrow \frac{\frac{2E}{1 + E^2} - \frac{2E}{1 + E^2}}{\frac{2E}{1 + E^2} - \frac{1}{\sqrt{1 + E^2}}} = \frac{0}{\frac{2E - \sqrt{1 + E^2}}{1 + E^2}} = 0$$

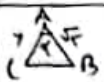
$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{1 + E^2}} \Rightarrow \frac{1}{10} - \frac{1}{10} = \frac{10 - 10}{10} = \frac{0}{10} = 0$$

(11)

$$r \sin \alpha < \frac{\sin^2 \alpha}{r \sin \alpha \cos \alpha} \Rightarrow r \sin \alpha - r \sin \alpha \cos \alpha < \frac{\sin^2 \alpha}{\sin \alpha \cos \alpha} \Rightarrow r \sin \alpha (1 - \cos \alpha) < \frac{\sin \alpha}{\cos \alpha}$$

$$\Rightarrow \frac{\cot \alpha}{\sin \alpha} > \frac{\cos \alpha}{\sin^2 \alpha} \Rightarrow \frac{\cos \alpha}{\sin^2 \alpha} > \frac{\cot \alpha}{\sin \alpha} \Rightarrow \frac{\cos \alpha}{\sin^2 \alpha} > \frac{\cos \alpha}{\sin \alpha} \Rightarrow \frac{1}{\sin^2 \alpha} > \frac{1}{\sin \alpha}$$

(12)



$$s = \frac{1}{r} b c \sin \alpha \Rightarrow \frac{1}{r} \times \sqrt{5}r \times r \times \sin \alpha = \frac{r^2}{r} \Rightarrow \sin \alpha = \frac{r}{r\sqrt{5}} = \frac{\sqrt{5}}{5} \Rightarrow \sin \alpha = \frac{\sqrt{5}}{5} \Rightarrow \alpha = 11.31^\circ, \dots$$

$$\Rightarrow \frac{r}{r} = \frac{r}{r}$$

(5) 1

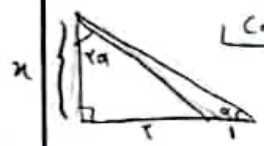


(طبق قضیه پیتاگوراس) $r = r\sqrt{1+r^2} = r\sqrt{2}$

$$r = \sqrt{r^2 + r^2} = r\sqrt{2} \Rightarrow \cos \alpha = \frac{r}{r\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \cos \alpha = \frac{\sqrt{2}}{2} \Rightarrow \alpha = 45^\circ$$

$$\Rightarrow \cot \alpha = 1$$

(5) 2

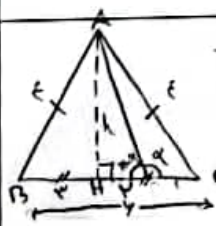


$\cot \alpha = ? \Rightarrow \cot \alpha = \frac{x}{1} = x$, $\cot 2\alpha = \frac{x}{x} = 1 \Rightarrow \cot 2\alpha = \frac{\cot \alpha - \tan \alpha}{1 + \cot \alpha \tan \alpha}$

$$\Rightarrow \cot \alpha = x + \tan \alpha \Rightarrow x = x^2 \left(1 + \frac{1}{x^2}\right) \Rightarrow x^2 = \frac{x}{2} \Rightarrow x = \frac{1}{2}$$

$$\Rightarrow \cot \alpha = \frac{x}{1} = \frac{1}{2}$$

(5) 3



ارتفاع مثلث مستوی الساقین همواره عمود بر پایه است و طبق قضیه پیتاگوراس داریم: $\tan \alpha = ?$

$$h = \sqrt{r^2 - \left(\frac{r}{2}\right)^2} = \frac{\sqrt{3}}{2}r \Rightarrow (r^2 = \left(\frac{r}{2}\right)^2 + h^2)$$

$$\Rightarrow \tan(\pi - \alpha) = -\tan \alpha = \frac{\sqrt{3}}{1} \Rightarrow \tan \alpha = -\frac{\sqrt{3}}{1}$$

(5) 4

$$\frac{r \sin^2 n + r \cos^2 n}{\sin^2 n + \cos^2 n} = \frac{r}{1} \Rightarrow \sin^2 n + 1 = \frac{r}{r} \Rightarrow \sin^2 n = \frac{1}{r} \Rightarrow \cos^2 n = 1 - \frac{1}{r} \Rightarrow \cos^2 n = \frac{r-1}{r}$$

$$\Rightarrow \tan^2 n = \frac{\sin^2 n}{\cos^2 n} = \frac{\frac{1}{r}}{\frac{r-1}{r}} = \frac{1}{r-1}$$

(5) 5