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سوال ۱)

$y = 1 - \log_c^{(ax-b)} \rightarrow \log_c^c - \log_c^{ax-b} \rightarrow y = \log_c^{\frac{c}{ax-b}}$

$\xrightarrow{(0,2)} \log_c^{\frac{-c}{b}} = 2 \rightarrow \frac{-c}{b} = c^2 \rightarrow \frac{-1}{b} = c \rightarrow b = \frac{-1}{c}$

$\rightarrow b + c = \frac{-3}{2} \rightarrow \frac{-1}{c} + c = \frac{-3}{2} \rightarrow 2c^2 - 2 = -3c \rightarrow 2c^2 + 3c - 2 = 0 \rightarrow c^2 + 3c - 4 = 0$ → ① $c = -1$ ، → $c = \frac{1}{4}$

غ ق ق چون نمودار صعودی است $\Leftarrow c > 1$

$b + \cancel{c}^{-1} = \frac{-3}{2} \rightarrow b = \frac{-1}{2}$

$\xrightarrow{(-1/25,0)} \log_{-1}^{\frac{-1}{-1/25a + \frac{1}{2}}} = 0 \rightarrow \frac{-1}{-1/25a + \frac{1}{2}} = 1 \rightarrow -1/25a + \frac{1}{2} = -1 \rightarrow -1/25a = -1/25 \rightarrow a = 1$

$(a+c)\,b = (1-1)^{-\frac{1}{2}} = 0$

سوال ۲)

$\xrightarrow{(0,\frac{2}{3})} 1 + c \times 3^a = \frac{2}{3} \rightarrow c \times 3^a = -\frac{1}{3}$ → $c = -1$ ، → $a = -1$

$\xrightarrow{(1,0)} 1 + -1 \times 3^{-1+b} = 0 \rightarrow b = 1$

$f(-1) = 1 + -1 \times 3^{-1-1} = 1 - \frac{1}{9} = \frac{8}{9}$

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سوال ۳)

$\xrightarrow{(0,2)} \begin{cases} c + \log_a^b = 2 \\ \xrightarrow{(2/4,0)} c + \log_a^{2/4a+b} = 0 \end{cases}$

$\log_a^{\frac{2/4a+b}{b}} = -2 \rightarrow \frac{2/4a+b}{b} = \frac{1}{25}$

$\rightarrow 40a + 25b = b \rightarrow 40a = -24b \rightarrow \frac{a}{b} = \frac{-24}{40} = -0/6$

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سوال ۴)

$|x^2-2| - x > 0 \rightarrow |x^2-2| > x \rightarrow x^2 - 2 > x \rightarrow x^2 - x - 2 > 0$ ، $x = -1, 2$

$\rightarrow x^2 - 2 < -x \rightarrow x^2 + x - 2 < 0$ ، $x = 1, -2$

	$-\infty$	-2	-1	1	$+\infty$
①		+	0	−	+
②		+	0	−	+

$D_f = ① \cap ② = (-2, -1)$ ، $D_f = (-\infty, -1) \cup (2, +\infty)$

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سوال ۵)

$x=1 \rightarrow -(1)^2 - 3(1) + 8 = 4$ → پس نقطه $(4,1)$ در هر دو تابع صدق می‌کند

$\xrightarrow{(1,4)} 2 + 2^{b-a} = 4 \rightarrow 2^{b-a} = 2 \rightarrow b - a = 1$

$f^{-1}(10) = -1 \rightarrow f(-1) = 10$

$\xrightarrow{(-1,10)} 2 + 2^{b+a} = 10 \rightarrow 2^{b+a} = 8 \rightarrow 2^{b+a} = 2^3 \rightarrow b + a = 3$

$b - a = 1$
$b + a = 3$
$b = 2$ ، $a = 1$

$2b - a = 4 - 1 = 3$

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فرناز محمدرضایی

سوال ۴)

$y = x^2 - x \xrightarrow{x=1} 1-1=0 \longrightarrow (1,0)$

$\xrightarrow{x=2} 4-2=2 \longrightarrow (2,2)$

} هر ۲ نقطه در تابع $f(x)$ صدق می‌کنند

$\xrightarrow{(1,0)} -2+\left(\frac{1}{2}\right)^{A+B} = 0 \longrightarrow 2^{-A-B} = 2 \longrightarrow -A-B=1$

$\xrightarrow{(2,2)} -2+\left(\frac{1}{2}\right)^{2A+B} = 2 \longrightarrow 2^{-2A-B} = 4 \longrightarrow 2^{-2A-B} = 2^2 \longrightarrow -2A-B=2$

$\begin{cases} -A-B=1 \\ -2A-B=2 \end{cases}$

$-A=1 \rightarrow \boxed{A=-1}$

$\boxed{B=0}$

$\xrightarrow{\$} f(x) = -2+\left(\frac{1}{2}\right)^{-x}$

$f(3) = -2+\left(\frac{1}{2}\right)^{-3} = -2+8 = \boxed{6}$

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سوال ۷)

$P_0 \times \left(\frac{8}{9}\right)^t = \frac{1}{4} P_0 \longrightarrow \left(\frac{8}{9}\right)^t = \frac{1}{4} \longrightarrow \log_{\frac{8}{9}}^{\left(\frac{8}{9}\right)^t} = \log_{\frac{8}{9}}^{4^{-1}} \longrightarrow t \log_{\frac{8}{9}}^{\frac{8}{9}} = -\log_{\frac{8}{9}}^{4}$

$\log_{\frac{8}{9}}^{\frac{8}{9}} = \log_{\frac{8}{9}}^{2^3} - \log_{\frac{8}{9}}^{3^2} \longrightarrow 3\log_{\frac{8}{9}}^{2} - 2\log_{\frac{8}{9}}^{3} = 3\left(\frac{10}{24}\right) - 2\left(\frac{10}{16}\right)$

$\log_{\frac{8}{9}}^{2} = \frac{1}{\log_2^{\frac{8}{9}}} = \frac{10}{24}$

$\log_{\frac{8}{9}}^{4} = \log_{\frac{8}{9}}^{2} + \log_{\frac{8}{9}}^{3} = \frac{10}{24} + \frac{10}{16}$

$\log_{\frac{8}{9}}^{3} = \frac{1}{\log_3^{\frac{8}{9}}} = \frac{10}{16}$

$3t\left(\frac{10}{24}\right) - 2t\left(\frac{10}{16}\right) = -\frac{10}{24} - \frac{10}{16} \longrightarrow (3t+1)\frac{10}{24} = (2t-1)\left(\frac{10}{16}\right)$

$\longrightarrow 24t - 12 = 21t + 7 \rightarrow 3t = 19 \rightarrow t = \frac{19}{3}\,h = 380\,min$

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سوال ۸)

$P_0 \times \left(\frac{875}{1000}\right)^t = \frac{1}{7} P_0 \longrightarrow \log_3^{\left(\frac{875}{1000}\right)^t} = \log_3^{7^{-1}} \longrightarrow t \log_3^{\frac{875}{1000}} = -\log_3^{7}$

$\log_3^{\frac{875}{1000}} = \log_3^{875} - \log_3^{1000} = \log_3^{5^3 \times 7} - \log_3^{2^3 \times 5^3} = 3\log_3^5 + \log_3^7 - 3\log_3^2 - 3\log_3^5$

$= \log_3^7 - 3\log_3^2$

$t \log_2^{\frac{875}{1000}} = -\log_2^7 \rightarrow t\left(\frac{10}{4}\right) - 3t\left(\frac{10}{16}\right) = -\frac{10}{4}$

$\log_3^7 = \frac{1}{\log_7^3} = \frac{10}{4}$

$\log_2^2 = \frac{1}{\log_3^2} = \frac{10}{16}$

$(t+1)\frac{10}{4} = 3t\left(\frac{10}{16}\right) \rightarrow 9t = 8t + 8$

$\longrightarrow t = 8_{\text{هفته}} = 56_{\text{روز}}$

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سوال ۹)

آب هر روز ۴ لیتر از محلول برداریم و جای آن آب بریزیم غلظت آن $\frac{1000-4}{1000} = \frac{24}{25}$ برابر می‌شود

$100 \times \left(\frac{24}{25}\right)^t = \frac{1}{2} \times 100 \longrightarrow \log\left(\frac{24}{25}\right)^t = \log \frac{1}{2}^{-1} \longrightarrow t \log \frac{24}{25} = -\log 2 \rightarrow -0.02t = -0.48 \rightarrow t = 24_{\text{روز}}$

$\log \frac{24}{25} = \log \frac{2^3 \times 3}{5^2} - \log 2 = \log 3 + 3\log 2 - 2\log \frac{10}{2} = \log 3 + 3\log 2 - 2\log 10 + 2\log 2 = -0.02$

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سوال ۱۰)

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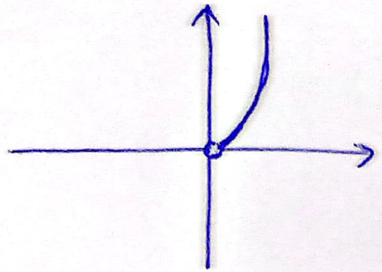
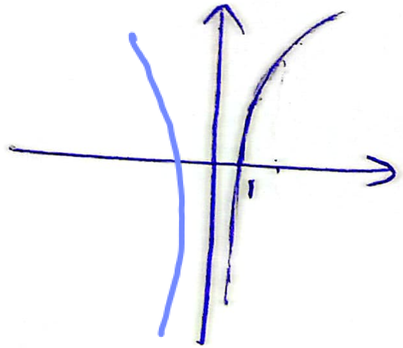
$D_f = (0, +\infty)$

الف) $y = 9^{\log_3^x} \rightarrow x^{\log_3^{3^2}} = y \rightarrow y = x^2$



ب) $y = \log x^2 = 2\log x$



1) $x=0 \rightarrow y=1-\log_c^{-b}=2 \rightarrow bc=-1$

$$\begin{cases} b+c=\frac{-3}{2} \\ bc=-1 \end{cases} \rightarrow \begin{cases} b=-2 \checkmark \\ b=\frac{1}{2} \times \end{cases}$$

b نمی‌تواند $(+1)$ باشد چون در این صورت c صفر می‌شود

$x=-1/5=\frac{-3}{2} \rightarrow 1-\log_{\frac{-1}{2}}^{\frac{-3}{2}a+2}=0 \rightarrow a=1 \quad (a+c)b=-3$