۱)

$$y=1-\log_c^{(ax-b)}=\log_c^{(ax-b)}-\log_c^{(ax-b)}=\log_c^{\frac{c}{ax-b}}$$

$$\xrightarrow{(0,2)} \log_c^{\frac{c}{a}}=2 \Rightarrow \frac{c}{a}=c^2 \xrightarrow{(0,0)} \frac{c}{a}=\frac{1}{b} \Rightarrow c=\frac{1}{b}$$

$$\left.\begin{array}{l} c=\frac{1}{b} \\ b+c=\frac{2}{3} \end{array}\right\} \Rightarrow \frac{1}{b}+b=\frac{2}{3} \xrightarrow{(\times b)} b^2-\frac{2}{3}b+1=0 \Rightarrow 3b^2-2b+3=0$$

$$\Delta=4-4\times3\times3=-32<0$$

$$\xrightarrow{(-10,0)} \log_{\frac{1}{2}}^{\frac{1}{a(-10)+b}}=0 \Rightarrow \frac{1}{-10a+b}=1 \Rightarrow -10a+b=2 \xrightarrow{(1,0)} a+b=\frac{1}{2}$$

$$\Rightarrow -9a=\frac{3}{2} \Rightarrow a=-\frac{1}{6} \Rightarrow b=\frac{2}{3}$$

$$\Rightarrow a+c=-\frac{1}{6}+\frac{1}{2}=\frac{1}{3}$$

۲)

$$f(x)=1+c\times a^{x^{a+b}}$$

$$\xrightarrow{(1,0)} 0=1+c\times a^{a+b} \Rightarrow c\times a^{a+b}=-1 \Rightarrow a^{a+b}=-1 \Rightarrow a=-1,\ b=1$$

$$\xrightarrow{(-2,\frac{1}{8})} \frac{1}{8}=1+c\times a^{b} \Rightarrow c\times a^{b}=-\frac{7}{8}$$

$$f(-1)=1+c\times a^{-b}=1+\frac{c}{a^b}=1+\frac{-\frac{7}{8}}{-1}=\frac{15}{8}$$

۳)

$$y=c+\log_5^{(ax+b)}$$

$$\xrightarrow{(2,2)} 2=c+\log_5^{b} \Rightarrow 2=-\log_5^{(2a+b)}+\log_5^{b} \Rightarrow \log_5^{\frac{b}{2a+b}}=2 \Rightarrow \frac{b}{2a+b}=25 \Rightarrow 25=50a+25b$$

$$\xrightarrow{(\frac{1}{10},0)} 0=c+\log_5^{\frac{1}{10}a+b} \Rightarrow c=-\log_5^{\frac{1}{10}a+b}$$

$$y=-\frac{\log_5^{\frac{1}{2}a+b}}{\log_5^{2a+b}}=-\frac{\log_5^{\frac{1}{2}a+b}}{\log_5^{2}+\log_5^{a+b}}=-\frac{\log_5^{\frac{1}{2}a+b}}{\log_5^{2}+\log_5^{\frac{1}{2}a+b}}$$

۴)

$$f(x)=\log_{\frac{1}{2}}^{(|x^2-1|-x)}$$

$$|x^2-1|-x>0 \Rightarrow |x^2-1|>x \Rightarrow \begin{cases} x^2-1>x \\ x^2-1<-x \end{cases}$$

(I) $x^2-x-1>0$, $\Delta=5$, $x=\frac{1\pm\sqrt{5}}{2}$

	$-\infty$		$\frac{1-\sqrt{5}}{2}$		$\frac{1+\sqrt{5}}{2}$		$+\infty$
	+	0	−	0	+		

(II) $x^2+x-1<0$, $\Delta=5$, $x=\frac{-1\pm\sqrt{5}}{2}$

	$-\infty$		$\frac{-1-\sqrt{5}}{2}$		$\frac{-1+\sqrt{5}}{2}$		$+\infty$
	+	0	−	0	+		

$$D_f=(I)\cup(II)=(-\infty,\frac{-1-\sqrt{5}}{2})\cup(\frac{1+\sqrt{5}}{2},+\infty)$$

۵)

$$x^3-3x^2+7x+2=ax^{b-a}+2^{b-a} \xrightarrow{x=1} 7=a+2^{b-a} \Rightarrow 2^{b-a}=7-a \Rightarrow b-a=\log_2^{7-a}$$

(I) $b-a=1 \Rightarrow 7-a=2 \Rightarrow a=5$

$$f^{-1}(1)=-1 \Rightarrow f(-1)=1 \Rightarrow -a+2^{b-a}=1 \Rightarrow -5+2^{1}=1 \Rightarrow b=6$$

(II) $a+b=11$

$$\Rightarrow (I)+(II)=2b=12 \Rightarrow b=6,\ a=5$$

$$(III)\ c=1-b=-5$$

۶)

$$-2+(\frac{1}{2})^{Ax+B}=x^2-x \xrightarrow{x=1} -2+(\frac{1}{2})^{A+B}=0 \Rightarrow 2^{-A-B}=2 \Rightarrow -A-B=1 \quad (I)$$

$$\xrightarrow{x=2} -2+(\frac{1}{2})^{2A+B}=2 \Rightarrow 2^{-2A-B}=4 \Rightarrow -2A-B=2 \quad (II)$$

$$(II)-(I) \Rightarrow -A=1 \Rightarrow A=-1$$

$$(I) \Rightarrow B=0$$

$$f(x)=-2+(\frac{1}{2})^{-x}=-2+2^{x}$$

$$f(x)=0 \Rightarrow 2^x=2 \Rightarrow x=1$$

۷)

$$(\frac{2}{3})^{t}=\frac{1}{4}\times\frac{9}{4} \Rightarrow (\frac{2}{3})^{t}=\frac{9}{16} \Rightarrow \log_{\frac{2}{3}}^{\frac{9}{16}}=t \xrightarrow{(I)} t=4 \Rightarrow h=\frac{81}{16}=5.0625$$

$$(I)\ \frac{81}{16}=\frac{9\times9}{4\times4}=\frac{\frac{81}{16}-\frac{32}{16}}{\frac{9}{4}\times\frac{4}{9}}=\frac{\frac{49}{16}}{1}=\frac{49}{16}$$

$$\log_{\frac{2}{3}}^{\frac{9}{4}}=\frac{\log_5^{3^2}-\log_5^{2^2}}{\log_5^{3^2}-\log_5^{2^3}}=\frac{2\log_5^{3}-2\log_5^{2}}{2\log_5^{3}-3\log_5^{2}}=\frac{-2\log_5^{\frac{3}{2}}}{-\log_5^{\frac{9}{8}}}=\frac{2\log_5^{\frac{3}{2}}}{\log_5^{\frac{9}{8}}}$$

$$\xrightarrow{\log_5^{\frac{9}{8}}=3\log_5^{\frac{3}{2}}} \frac{2}{3}$$

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$\log_{\frac{1}{2}}\left(\frac{x-1}{x+5}\right)^2 = \log_{\frac{1}{2}}\left(\frac{x}{8}\right)^2 \Rightarrow \left(\frac{x-1}{x+5}\right)^2 = \left(\frac{x}{8}\right)^2 \Rightarrow \frac{x-1}{x+5} = \frac{x}{8} \Rightarrow x^2+5x = 8x-8 \Rightarrow x^2-3x+8=0$

$\Delta = 9-32 = -23 < 0$ (ت) $\frac{-1}{\frac{1}{24}+\frac{1}{8}} = \frac{-24}{4} = -6$

$\log_{\frac{1}{2}}\frac{1}{8} - \log_{\frac{1}{2}}\frac{1}{3} = \frac{1}{\log_{\frac{1}{2}}\frac{1}{8}} - \frac{1}{\log_{\frac{1}{2}}\frac{1}{3}} = \frac{\log_{\frac{1}{2}}3 - \log_{\frac{1}{2}}8}{\log_{\frac{1}{2}}3 \times \log_{\frac{1}{2}}8} = \frac{-3+1}{(-\log_2 3)(-3)} = \frac{-2}{3\log_2 3}$

$\log_{\frac{1}{2}}\frac{1}{8} = \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^3 = 3$

$\frac{-2}{3\log_2 3} \times 3 = \frac{-2}{\log_2 3} = \log_3 2$

۸

$\frac{x-1}{x+5} = \frac{x}{8} \Rightarrow x^2+5x = 8x-8 \Rightarrow x^2-3x+8=0$

$\Delta = 9-32 = -23 < 0$ (ت) $\frac{-1}{\frac{1}{24}+\frac{1}{8}} = \frac{-24}{4} = -6$

$\log_{\frac{1}{2}}\frac{1}{8} - \log_{\frac{1}{2}}\frac{1}{3} = \frac{1}{\log_{\frac{1}{2}}\frac{1}{8}} - \frac{1}{\log_{\frac{1}{2}}\frac{1}{3}} = \frac{\log_{\frac{1}{2}}3 - \log_{\frac{1}{2}}8}{\log_{\frac{1}{2}}3 \times \log_{\frac{1}{2}}8} = \frac{-3+1}{(-\log_2 3)(-3)} = \frac{-2}{3\log_2 3}$

$\log_{\frac{1}{2}}\frac{1}{8} = \log_{\frac{1}{2}}\left(\frac{1}{2}\right)^3 = 3$

$\frac{-2}{3\log_2 3} \times 3 = \frac{-2}{\log_2 3} = \log_3 2$

۹

$\frac{x-100}{x} = \frac{25}{24} \Rightarrow 1 - \frac{100}{x} = \frac{25}{24} \Rightarrow \frac{100}{x} = -\frac{1}{24} \Rightarrow x = -2400$

$\log_{\frac{1}{2}}\left(\frac{25}{24}\right)^2 = \log_{\frac{1}{2}}\frac{1}{2} = -1$ (ب $\log_{\frac{25}{24}}\frac{25}{24} = 1$

$\log_{\frac{25}{24}}\frac{25}{24} = -\log_{\frac{25}{24}}\frac{24}{25} \rightarrow -\log_{\frac{25}{24}}\frac{25}{24} = -1$

روش دوم

۱۰

الف) $y = 9^{\log_3 x} \Rightarrow x > 0$

$y = x^{\log_3 9} \Rightarrow y = x^2$

$D_y = (0, +\infty)$

ب) $y = \log x^2 \Rightarrow x^2 > 0 \Rightarrow x \neq 0$

$= 2\log|x|$

$D_y = \mathbb{R} - \{0\}$