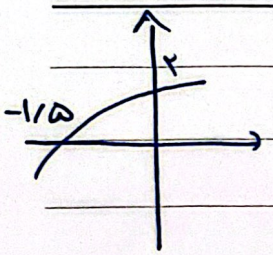


① $(0,2) \Rightarrow 2 = 1 - \log_c^{(0-b)}$

$(-1/5, 0) \Rightarrow 0 = 1 - \log_c(-1/5a - b)$

$\Rightarrow -1 = \log_c^{(-b)} \Rightarrow \frac{1}{c} = -b \Rightarrow b = -\frac{1}{c} \Rightarrow b = \frac{-1}{\frac{1}{2}} = -2$

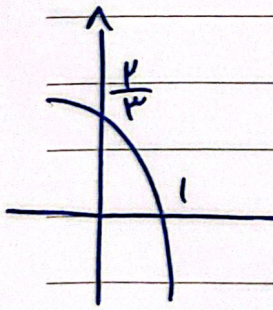
$1 = \log_c(-1/5a - b)$

$\frac{-1}{c} + c = \frac{-3}{2} \Rightarrow \frac{-1+c^2}{c} = -\frac{3}{2} \Rightarrow 2c^2 + 3c - 2 = 0$

$\Rightarrow c^2 + 3c - 4 = 0 \Rightarrow (c+4)(c-1) = 0 \Rightarrow \begin{cases} c = -4 \rightarrow \text{غ ق ق} \\ c = +\frac{1}{2} \Rightarrow b = -2 \end{cases}$

$+1 = \log_2(-1/5a + 2) \Rightarrow -1/5a + 2 = 2 \Rightarrow -1/5a = 0 \Rightarrow a = 0$

$(0 + \frac{1}{2}) \times -2 = \boxed{-1}$



② $f(x) = 1 + c \times 3^{ax+b}$ $\quad f(-1) = ?$

$(I)(1,0) \Rightarrow 0 = 1 + c \times 3^{a+b}$

$(II)(0, \frac{2}{3}) \Rightarrow \frac{2}{3} = 1 + c \times 3^{a}$

$\Rightarrow \frac{2}{3} = c(3^a - 3^{a+b}) \Rightarrow c = \frac{2}{3(3^a - 3^{a+b})}$

صورت‌های دو معادله $\Rightarrow c = \frac{2}{3^a(3 - 3^{b+1})}$ $\quad (II)$

$\frac{2}{3} = 1 + \frac{2}{3^a(3-3^{b+1})} \times 3^a \Rightarrow -\frac{1}{3} = \frac{2}{(3-3^{b+1})} \Rightarrow$

$3 - 3^{b+1} = -6 \Rightarrow 3^{b+1} = 3^2 \Rightarrow b+1 = 2 \Rightarrow b = 1$

$f(-1) \Rightarrow y = 1 + c(3^{a-1})$

$f(1) \Rightarrow 0 = 1 + c(3^{a+1})$

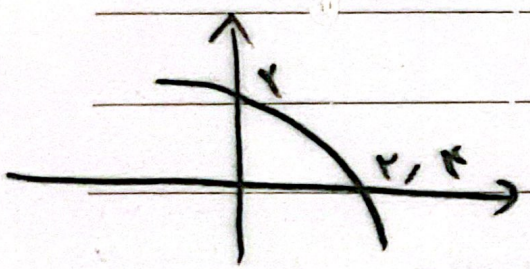
$\div \Rightarrow \frac{-1}{y-1} = \frac{c \times 3^{a+1}}{c \times 3^{a-1}}$

$\Rightarrow \frac{-1}{y-1} = 3^2 \Rightarrow -1 = 9y - 9 \Rightarrow 9y = 8 \Rightarrow y = \frac{8}{9}$

$\Rightarrow f(-1) = \frac{8}{9}$

(۳)

$y = C + \log_a^{(ax+b)}$ $\quad \frac{a}{b} = ?$



$(۲٫۴, ۰) \Rightarrow ۰ = C + \log_a^{(۲٫۴a+b)}$

$(۰, ۲) \Rightarrow ۲ = C + \log_a^{(b)}$

$۲ = \log_a^{b} - \log_a^{(۲٫۴a+b)} \Rightarrow ۲ = \log_a^{\frac{b}{۲٫۴a+b}} \Rightarrow \frac{b}{۲٫۴a+b} = \underbrace{a^{۲}}_{۲a} \Rightarrow ۶۰a + ۲۴b = b$

$\Rightarrow ۶۰a = -۲۴b \Rightarrow \frac{a}{b} = \frac{-۲۴}{۶۰} = \boxed{\frac{-۲}{۵}}$

(۴)

$f(x) = \log_{۴}^{(|x^۲-۲|-x)}$

(I) $x^۲ - ۲ \geq ۰ \Rightarrow x^۲ \geq ۲ \Rightarrow x \geq \sqrt{۲}$ یا $x \leq -\sqrt{۲}$

$\Rightarrow x^۲ - ۲ - x > ۰ \Rightarrow x < -۱$ یا $x > ۲$ (II) $\xrightarrow{\text{(I) و (II)}}$ $x \leq -\sqrt{۲}$ یا $x > ۲$ (۱)

(III) $x^۲ - ۲ < ۰ \Rightarrow x^۲ < ۲ \Rightarrow -\sqrt{۲} < x < \sqrt{۲} \Rightarrow -x^۲ + ۲ - x > ۰$

$\Rightarrow -۲ < x < ۱$ (IV) $\xrightarrow{\cap}$ $-\sqrt{۲} < x < ۱$ (۲) $\xrightarrow{(۱) \cup (۲)}$ $D_f = (-\infty, ۱) \cup (۲, +\infty)$

$f(x) = 2 + 2^{b-ax}$ $\quad f^{-1}(10) = -1 \quad 2b-a = ?$ (a)

$g(x) = -x^2 - 2x + 8$ $\quad f(-1) = 10 \Rightarrow f(-1) = 2 + 2^{b+a}$

$\Rightarrow 2 + 2^{b+a} = 10 \Rightarrow 2^{b+a} = 8 \Rightarrow 2^{b+a} = 2^3$

$f(1) = g(1)$ $\quad b + a = 3 \ (I)$

$\Rightarrow 2 + 2^{b-a} = \frac{-1-2+8}{4} \Rightarrow 2^{b-a} = 2 \Rightarrow b - a = 1 \ (II)$

$\xrightarrow{(I),(II)} \begin{cases} b+a=3 \\ b-a=1 \end{cases}$

$2b = 4 \Rightarrow b = 2, \ a = 1 \Rightarrow 2b - a = 4 - 1 = 3$

$f(x) = -2 + \left(\frac{1}{2}\right)^{Ax+B}$ $\quad y = x^2 - x$ (c)

$f(y) = 6$

$f(1) = -2 + \left(\frac{1}{2}\right)^{A+B}$ $\quad y = 1 - 1 = 0 \ (I)$

$f(2) = -2 + \left(\frac{1}{2}\right)^{2A+B}$ $\quad y = 4 - 2 = 2 \ (II)$

$(I) \Rightarrow -2 + \left(\frac{1}{2}\right)^{A+B} = 0 \Rightarrow \left(\frac{1}{2}\right)^{A+B} = 2 \Rightarrow 2^{-A-B} = 2$

$\Rightarrow -(A+B) = 1 \Rightarrow A + B = -1 \ (1)$

$(II) \Rightarrow -2 + \left(\frac{1}{2}\right)^{2A+B} = 2 \Rightarrow \left(\frac{1}{2}\right)^{2A+B} = 4 \Rightarrow$

$2^{-2A-B} = 2^2 \Rightarrow -(2A+B) = 2 \Rightarrow 2A + B = -2 \ (2)$

$\xrightarrow{(1),(2)} \begin{cases} A + B = -1 \xrightarrow{\times(-)} \\ 2A + B = -2 \end{cases} \Rightarrow \begin{cases} -A - B = 1 \\ 2A + B = -2 \end{cases}$

$A = -1, \ B = 0$

$\Rightarrow f(x) = -2 + \left(\frac{1}{2}\right)^{-x} \Rightarrow f(y) = -2 + \left(\frac{1}{2}\right)^{-3} = -2 + 2^3$

$= -2 + 8 = 6$

۷) جرم عنصر بعد از $\frac{t}{4}$ ساعت برابر می‌شود با $\frac{1}{9}$ ⇐ $(\frac{1}{9})^{\frac{t}{4}} = \frac{1}{6}$

$$\log_5^{(\frac{1}{9})^{\frac{t}{4}}} = \log_5^{(\frac{1}{6})} \Rightarrow \frac{t}{4}\log_5^{(\frac{1}{9})} = -1\log_5^6$$

$$\log_5^3 = 1.4 = \frac{14}{10} = \frac{7}{5} \Rightarrow \log_5^3 = \frac{5}{7}$$

$$\log_2^6 = 2.4 = \frac{24}{10} = \frac{12}{5} \Rightarrow \log_5^2 = \frac{5}{12}$$

$$\frac{t}{4}\log_5^{(\frac{1}{9})} = -\log_5^6 \Rightarrow \frac{t}{4}(\underbrace{\log_5^8}_{3\log_5^2} - \underbrace{\log_5^9}_{2\log_5^3}) = -(\log_5^2 + \log_5^3)$$

$$\Rightarrow \frac{t}{4}(3\times\frac{5}{12} - 2\times\frac{5}{7}) = -(\frac{5}{12} + \frac{5}{7}) \Rightarrow \frac{t}{4}(\frac{35-40}{28}) = -(\frac{35+40}{84})$$

$$\Rightarrow -5t = -\frac{95}{3} \Rightarrow t = \frac{19}{3} \xrightarrow[\text{دقیقه}]{\times 60} \boxed{\text{دقیقه } 380}$$

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۸) جرم عنصر در هفته $\frac{7}{8}$ برابر می‌شود ⇐ $(\frac{7}{8})^{\frac{t}{7}} = \frac{1}{7}$

$$\log_3^{(\frac{7}{8})^{\frac{t}{7}}} = \log_3^{(\frac{1}{7})} \Rightarrow \frac{t}{7}\log_3^{(\frac{7}{8})} = \log_3^{(\frac{1}{7})}$$

$$\Rightarrow \frac{t}{7}(\log_3^7 - \log_3^8) = -\log_3^7$$

$$\log_7^3 = 0.6 = \frac{6}{10} = \frac{3}{5} \Rightarrow \log_3^7 = \frac{5}{3}$$

$$\log_2^3 = 1.6 = \frac{16}{10} = \frac{8}{5} \Rightarrow \log_3^2 = \frac{5}{8}$$

$$\Rightarrow \frac{t}{7}(\log_3^7 - 3\log_3^2) = -\log_3^7 \Rightarrow \frac{t}{7}(\frac{5}{3} - 3\times\frac{5}{8}) = -\frac{5}{3}$$

$$\Rightarrow \boxed{t = 56}$$

(۹) عفونت محلول در هر روز $\frac{۹۶}{۱۰۰}$ برابر می‌شود

عفونت اولیه = A

$$f(t) = A\left(\frac{۹۶}{۱۰۰}\right)^t \Rightarrow \frac{A}{۳} = A\left(\frac{۹۶}{۱۰۰}\right)^t \Rightarrow \left(\frac{۹۶}{۱۰۰}\right)^t = \frac{۱}{۳}$$

$$\Rightarrow \log\left(\frac{۹۶}{۱۰۰}\right)^t = \log\frac{۱}{۳} \Rightarrow t(\log ۹۶ - \log ۱۰۰) = -\log ۳$$

$۹۶ = ۲^۵ \times ۳$ $\Rightarrow t(\log ۲^۵ + \log ۳ - ۲) = -\log ۳ \Rightarrow$

$$t(۵(۰/۳) + ۰/۴۸ - ۲) = -۰/۴۸ \Rightarrow t(۱/۵ + ۰/۴۸ - ۲) = -۰/۴۸$$

$$\Rightarrow t = ۲۴$$

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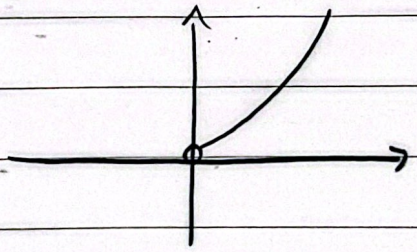
(۱۰) الف) $y = ۹^{\log_۳ x}$ $\Rightarrow y = x^{\log_۳ ۹}$ $\Rightarrow y = x^۲$

$x > ۰$ $\Rightarrow$



ب) $y = \log x^۲ \Rightarrow y = ۲\log x \Rightarrow$

x	$\frac{۱}{۲}$	۱
y	۰