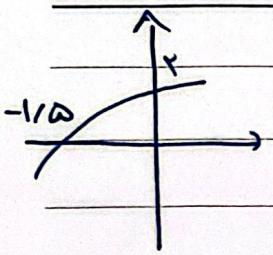


① $(0,2) \Rightarrow 2 = 1 - \log_c^{(0-b)}$

$(-1/25, 0) \Rightarrow 0 = 1 - \log_c^{(-1/25a - b)}$

$\Rightarrow -1 = \log_c^{(-b)} \Rightarrow \frac{1}{c} = -b \Rightarrow b = -\frac{1}{c} \Rightarrow b = \frac{-1}{\frac{1}{2}} = -2$

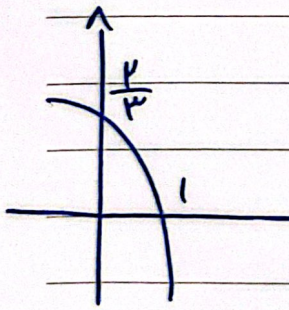
$1 = \log_c^{(-1/25a - b)}$

$\frac{-1}{c} + c = \frac{-3}{2} \Rightarrow \frac{-1+c^2}{c} = -\frac{3}{2} \Rightarrow 2c^2 + 3c - 2 = 0$

$\Rightarrow c^2 + 3c - 4 = 0 \Rightarrow (c+4)(c-1) = 0 \Rightarrow \begin{cases} c = -4 \text{ غ ق ق} \\ c = +\frac{1}{2} \Rightarrow b = -2 \end{cases}$

$+1 = \log_2^{(-1/25a + 2)} \Rightarrow -1/25a + 2 = 2 \Rightarrow -1/25a = 0 \Rightarrow a = 0$

$(0 + \frac{1}{2}) \times -2 = \boxed{-1}$



② $f(x) = 1 + c \times 3^{ax+b}$ $\quad f(-1) = ?$

$(I)(1,0) \Rightarrow 0 = 1 + c \times 3^{a+b}$

$(II)(0, \frac{2}{3}) \Rightarrow \frac{2}{3} = 1 + c \times 3^{a}$

$\Rightarrow \frac{2}{3} = c(3^a - 3^{a+b}) \Rightarrow c = \frac{2}{3(3^a - 3^{a+b})}$

③ جایگذاری در معادله (II)

$\Rightarrow c = \frac{2}{3^a(3 - 3^{b+1})}$

$\frac{2}{3} = 1 + \frac{2}{3^a(3-3^{b+1})} \times 3^a \Rightarrow -\frac{1}{3} = \frac{2}{(3-3^{b+1})} \Rightarrow$

$3 - 3^{b+1} = -6 \Rightarrow 3^{b+1} = 3^2 \Rightarrow b+1 = 2 \Rightarrow b = 1$

$f(-1) \Rightarrow y = 1 + c(3^{a-1})$

$f(1) \Rightarrow 0 = 1 + c(3^{a+1})$

$\Rightarrow \frac{-1}{y-1} = \frac{c \times 3^{a+1}}{c \times 3^{a-1}}$

$\Rightarrow \frac{-1}{y-1} = 3^2 \Rightarrow -1 = 9y - 9 \Rightarrow 9y = 8 \Rightarrow y = \frac{8}{9}$

$\Rightarrow \boxed{f(-1) = \frac{8}{9}}$

dotnote
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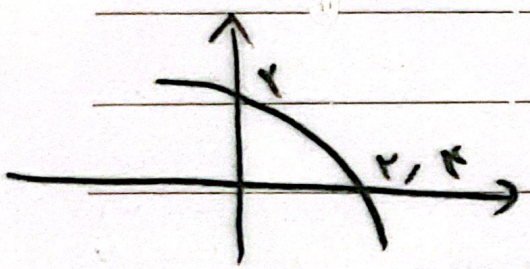

$y = C + \log_a^{(ax+b)}$ $\quad \frac{a}{b} = ?$ (۳)

$(۲, ۲۴, ۰) \Rightarrow ۰ = C + \log_a^{(۲/۴a+b)}$

$(۰, ۲) \Rightarrow ۲ = C + \log_a^{(b)}$

$۲ = \log_a^b - \log_a^{(۲/۴a+b)} \Rightarrow ۲ = \log_a^{\frac{b}{۲/۴a+b}} \Rightarrow \frac{b}{۲/۴a+b} = \underbrace{a^۲}_{۲a} \Rightarrow ۶۰a + ۲۴b = b$

$\Rightarrow ۶۰a = -۲۴b \Rightarrow \frac{a}{b} = \frac{-۲۴}{۶۰} = \boxed{\frac{-۲}{۵}}$

$f(x) = \log_۴^{(|x^۲-۲|-x)}$ (۴)

(I) $x^۲ - ۲ \geq ۰ \Rightarrow x^۲ \geq ۲ \Rightarrow x \geq \sqrt{۲}$ یا $x \leq -\sqrt{۲}$

$\Rightarrow x^۲ - ۲ - x > ۰ \Rightarrow x < -۱$ یا $x > ۲$ (II) $\xrightarrow{\text{(I)} \cap \text{(II)}}$ $x \leq -\sqrt{۲}$ یا $x > ۲$ (۱)

(III) $x^۲ - ۲ < ۰ \Rightarrow x^۲ < ۲ \Rightarrow -\sqrt{۲} < x < \sqrt{۲} \Rightarrow -x^۲ + ۲ - x > ۰$

$\Rightarrow -۲ < x < ۱$ (IV) $\xrightarrow{\cap}$ $-\sqrt{۲} < x < ۱$ (۲) $\xrightarrow{(۱) \cup (۲)}$ $D_f = (-\infty, ۱) \cup (۲, +\infty)$

dotnote

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$f(x) = 2 + 2^{b-ax}$ $\quad f^{-1}(10) = -1 \quad 2b-a = ?$ ④

$g(x) = -x^2 - 2x + 8$ $\quad f(-1) = 10 \Rightarrow f(-1) = 2 + 2^{b+a}$

$\Rightarrow 2 + 2^{b+a} = 10 \Rightarrow 2^{b+a} = 8 \Rightarrow 2^{b+a} = 2^3$

$f(1) = g(1)$ $\quad b + a = 3 \ (I)$

$\Rightarrow 2 + 2^{b-a} = \frac{-1-2+8}{4} \Rightarrow 2^{b-a} = 2 \Rightarrow b - a = 1 \ (II)$ ⑥

$(I), (II) \rightarrow \begin{cases} b+a=3 \\ b-a=1 \end{cases}$

$2b = 4 \Rightarrow b = 2, \ a = 1 \Rightarrow 2b - a = 4 - 1 = 3$

$f(x) = -2 + \left(\frac{1}{2}\right)^{Ax+B}$ $\quad y = x^2 - x$ ⑤

$f(y) = 6$

$f(1) = -2 + \left(\frac{1}{2}\right)^{A+B}$ $\quad y = 1 - 1 = 0 \ (I)$

$f(2) = -2 + \left(\frac{1}{2}\right)^{2A+B}$ $\quad y = 4 - 2 = 2 \ (II)$ ⑤

$(I) \Rightarrow -2 + \left(\frac{1}{2}\right)^{A+B} = 0 \Rightarrow \left(\frac{1}{2}\right)^{A+B} = 2 \Rightarrow 2^{-A-B} = 2$

$\Rightarrow -(A+B) = 1 \Rightarrow A + B = -1 \ (1)$

$(II) \Rightarrow -2 + \left(\frac{1}{2}\right)^{2A+B} = 2 \Rightarrow \left(\frac{1}{2}\right)^{2A+B} = 4 \Rightarrow$

$2^{-2A-B} = 2^2 \Rightarrow -(2A+B) = 2 \Rightarrow 2A + B = -2 \ (2)$

$(1), (2) \rightarrow \begin{cases} A + B = -1 \xrightarrow{\times(-)} \\ 2A + B = -2 \end{cases} \Rightarrow \begin{cases} -A - B = 1 \\ 2A + B = -2 \end{cases}$

$A = -1, \ B = 0$

$\Rightarrow f(x) = -2 + \left(\frac{1}{2}\right)^{-x} \Rightarrow f(y) = -2 + \left(\frac{1}{2}\right)^{-3} = -2 + 2^3$

$= -2 + 8 = 6$

۷) جرم عنصر بعد از $\frac{t}{4}$ ساعت برابر می‌شود با $\frac{1}{9}$ ⇐ $(\frac{1}{9})^{\frac{t}{4}} = \frac{1}{6}$

$\log_{\frac{1}{5}}^{(\frac{1}{6})^{\frac{t}{4}}} = \log_{\frac{1}{5}}^{(\frac{1}{6})} \Rightarrow \frac{t}{4}\log_{\frac{1}{5}}^{(\frac{1}{6})} = -1\log_{\frac{1}{5}}^{6}$

$\log_{\frac{3}{5}}^{5} = 1.4 = \frac{14}{10} = \frac{7}{5} \Rightarrow \log_{\frac{3}{5}}^{\frac{5}{7}}$ (3)

$\log_{\frac{2}{5}}^{9} = 2.4 = \frac{24}{10} = \frac{12}{5} \Rightarrow \log_{\frac{2}{5}}^{\frac{5}{12}}$

$\frac{t}{4}\log_{\frac{1}{5}}^{(\frac{1}{6})} = -\log_{\frac{1}{5}}^{6} \Rightarrow \frac{t}{4}(\underbrace{\log_{\frac{1}{5}}^{8}}_{3\log_{\frac{1}{5}}^{2}} - \underbrace{\log_{\frac{1}{5}}^{9}}_{2\log_{\frac{1}{5}}^{3}}) = -(\log_{\frac{1}{5}}^{2} + \log_{\frac{1}{5}}^{3})$

$\Rightarrow \frac{t}{4}(3\times\frac{5}{12} - 2\times\frac{5}{7}) = -(\frac{5}{12} + \frac{5}{7}) \Rightarrow \frac{t}{4}(\frac{35-40}{28}) = -(\frac{35+40}{84})$

$\Rightarrow -5t = -\frac{95}{3} \Rightarrow t = \frac{19}{3} \xrightarrow[\text{دقیقه}]{\times 60} 380$ دقیقه

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۸) جرم عنصر در هفته $\frac{7}{8}$ برابر می‌شود ⇐ $(\frac{7}{8})^{\frac{t}{7}} = \frac{1}{7}$

$\log_{3}^{(\frac{7}{8})^{\frac{t}{7}}} = \log_{3}^{(\frac{1}{7})} \Rightarrow \frac{t}{7}\log_{3}^{(\frac{7}{8})} = \log_{3}^{(\frac{1}{7})}$

$\Rightarrow \frac{t}{7}(\log_{3}^{7} - \log_{3}^{8}) = -\log_{3}^{7}$ (6)

$\log_{7}^{3} = 0.6 = \frac{6}{10} = \frac{3}{5} \Rightarrow \log_{3}^{7} = \frac{5}{3}$

$\log_{2}^{3} = 1.6 = \frac{16}{10} = \frac{8}{5} \Rightarrow \log_{3}^{2} = \frac{5}{8}$

$\Rightarrow \frac{t}{7}(\log_{3}^{7} - 3\log_{3}^{2}) = -\log_{3}^{7} \Rightarrow \frac{t}{7}(\frac{5}{3} - 3\times\frac{5}{8}) = -\frac{5}{3}$

$\Rightarrow t = 56$

dotnote

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۹) غفلت محلول در هر روز $\frac{۹۶}{۱۰۰}$ برابر می‌شود

غفلت اولیه = A

$$f(t) = A\left(\frac{۹۶}{۱۰۰}\right)^t \Rightarrow \frac{A}{۲} = A\left(\frac{۹۶}{۱۰۰}\right)^t \Rightarrow \left(\frac{۹۶}{۱۰۰}\right)^t = \frac{۱}{۲}$$

$$\Rightarrow \log\left(\frac{۹۶}{۱۰۰}\right)^t = \log\frac{۱}{۲} \Rightarrow t(\log ۹۶ - \log ۱۰۰) = -\log ۲$$

$۹۶ = ۲^۵ \times ۳$ ⟹ $t(\log ۲^۵ + \log ۳ - ۲) = -\log ۳ \Rightarrow$ (۵)

$$t(۵(۰/۳) + ۰/۴۸ - ۲) = -۰/۴۸ \Rightarrow t(۱/۵ + ۰/۴۸ - ۲) = -۰/۴۸$$

$$\Rightarrow \boxed{t = ۲۴}$$

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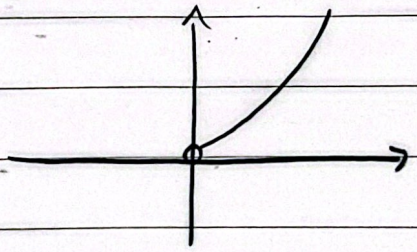
۱۰) الف) $y = ۹^{\log_۳ x^۲} \Rightarrow y = x^{\log_۹ ۲} \Rightarrow y = x^۲$

$x > ۰$ ⟹

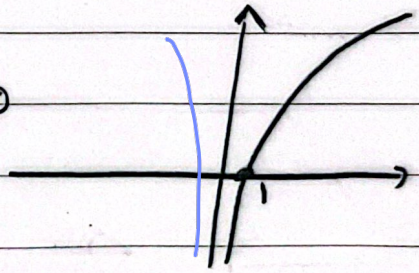


(۱,۷۵)

ب) $y = \log x^۲ \Rightarrow y = ۲\log x \Rightarrow$

x	$\frac{۱}{۲}$	۱
y	۰	

$D = \mathbb{R} - \{۰\}$



۱) $x = ۰ \rightarrow y = ۱ - ۷y_c^{-b} = ۲ \rightarrow bc = -۱$

$$\begin{cases} b + c = -\frac{۳}{۲} \\ bc = -۱ \end{cases} \rightarrow \begin{cases} b = -۲ \checkmark \\ b = \frac{۱}{۲} \times \end{cases}$$

b نمی‌تواند (+) باشد چون در این صورت c صفر می‌شود

$x = -۱, a = -\frac{۳}{۲} \rightarrow ۱ - ۷y_{-\frac{۱}{۲}}^{-\frac{۳}{۲}a+۲} = ۰ \rightarrow a = ۱ \quad (a+c)b = -۳$