لیریا احینی

۱-

$f(x) = 3^{Ax+B}$ $\quad y = x^2$

$x=1 \Rightarrow y = f(x) \Rightarrow 3^{A+B} = 1 \Rightarrow A+B=0 \Rightarrow A=-B$

$x=3 \Rightarrow y = f(x) \Rightarrow 3^{3A+B} = 3^2 = 9 \Rightarrow 3A+B=2 \xrightarrow{} -3B+B=2 \Rightarrow -2B=2 \Rightarrow B=-1$

$A=-B=1$

$f(x) = 3^{x-1}$

عرض از مبدا $\Rightarrow x=0 \Rightarrow 3^{0-1} = 3^{-1} = \frac{1}{3} = y'$ تقاطع با محور yها

۲-

$\log_2^{4^x+15} = x+3 \Rightarrow 2^{x+3} = 4^x + 15 \Rightarrow 2^x \times 2^3 - (2^x)^2 - 15 = 0$ $\quad (2^x)^2 = (2^2)^x = 4^x$

$t = 2^x \Rightarrow 8t - t^2 - 15 = 0 \Rightarrow t^2 - 8t + 15 = 0 \Rightarrow (t-3)(t-5) = 0 \begin{cases} t=3 \\ t=5 \end{cases}$

$t = 2^x = 3 \Rightarrow x = \log_2^3$

$t = 2^x = 5 \Rightarrow x = \log_2^5$

$\Big\}$ مجموع $= \log_2^3 + \log_2^5 = \log_2^{3\times5} = \log_2^{15}$

۳-

$(\log_{21}^3)^2 + \log_{21}^{(147)} \log_{21}^{(1323)}$

$147 = 3 \times 7 \times 7 = \frac{3 \times 7 \times 3 \times 7}{3} = \frac{21^2}{3}$ $\quad \log_{21}^{147} = \log_{21}^{21^2} - \log_{21}^3 = \underbrace{2\log_{21}^{21}}_{2} - \log_{21}^3$

$= 2 - \log_{21}^3$

$1323 = 7 \times 7 \times 3 \times 3 \times 3 = 21^2 \times 3$

$\log_{21}^{1323} = \log_{21}^{21^2} + \log_{21}^3 = 2\log_{21}^{21} + \log_{21}^3 = 2 + \log_{21}^3$

$(\log_{21}^3)^2 + \underbrace{(2 - \log_{21}^3)(2 + \log_{21}^3)}_{\text{مزدوج}} = (\log_{21}^3)^2 + 4 - (\log_{21}^3)^2 = 4$

دریا احمدی

۴-

$\log(x^2-2x+1)+3\log(1-x)=0$

$\log\underbrace{(x-1)^2}_{(1-x)^2}+3\log(1-x)=0 \rightarrow 2\log(1-x)+3\log(1-x)=0$

$\log(1-x)=0 \rightarrow \log_{10}^{1-x}=1 \Rightarrow 1-x=10 \rightarrow x=-9$

$\log_3^{-(-9)}=\log_3^9=\log_3^{3^2}=2\log_3^3=2$

۵-

$\log_2^{(x^2+2x+3)}+\log_2^{(x-2)}=3$

$\log_2^{\overbrace{(x^2+2x+3)(x-2)}^{\text{فیک وفنیون}}}=3 \rightarrow \log_2^{x^3-8}=3 \rightarrow 2^3=x^3-8 \rightarrow$

$14=x^3 \rightarrow x=\sqrt[3]{14}$

$\log_{\sqrt[3]{2}}^{\sqrt[3]{14}}=\log_{2^{\frac{1}{3}}}^{14^{\frac{1}{3}}}=\frac{\frac{1}{3}}{\frac{1}{3}}\log_2^{14}=\log_2^{2^4}=4\log_2^2=4$

۶-

$\log(2-x)-\log\frac{1}{\underbrace{(x-2)^2}_{(2-x)^2}}=3 \quad t=2-x \rightarrow \log t-\log\frac{1}{t^2}=3 \rightarrow$

$\log\frac{t}{\frac{1}{t^2}}=3 \rightarrow \log_{10}^{t^3}=3 \rightarrow 1000=t^3 \rightarrow t=10 \rightarrow 2-x=t=10 \rightarrow x=-8$

$\log_{\sqrt{2}}^{(-x)}=\log_{\sqrt{2}}^{-(-8)}=\log_{2^{\frac{1}{2}}}^{2^3}=\frac{3}{\frac{1}{2}}\log_2^2=6$

دبیر: امینی

۷-

$81^n = 3^{(n^2-2)} \rightarrow 3^{4n} = 3^{n^2-2} \rightarrow 4n = n^2-2 \rightarrow n^2-4n-2=0$

$(3^4)^n$

$\Delta = 16-4(1)(-2) = 16+8 = 24$

$n = \frac{4 \pm \sqrt{24}}{2} = \frac{4 \pm 2\sqrt{6}}{2} = \frac{2 \pm \sqrt{6}}{1}$ $\begin{cases} \rightarrow 2+\sqrt{6} \checkmark \\ \rightarrow 2-\sqrt{6} \text{ غ ق ق} \end{cases}$

مبنای log را عضفی می کند

$\log_4^{n-2} = \log_4^{2+\sqrt{6}-2} = \log_4^{\sqrt{6}} = \log_4^{4^{\frac{1}{2}}} = \frac{1}{2}\log_4^4 = \frac{1}{2}$

۸-

$\log_3^2 = \frac{a}{8}$

$\log_{18}^8 = \frac{\log_3^8}{\log_3^{18}}$

$\log_3^8 = \log_3^{2^3} = 3\log_3^2 = 3\times\frac{a}{8}$

$\log_3^{18} = \log_3^{9\times2} = \log_3^9 + \log_3^2 = \log_3^{3^2} + \log_3^2 = 2\log_3^3 + \frac{a}{8} = \frac{16}{8} + \frac{a}{8} = \frac{21}{8}$

$\frac{\log_3^8}{\log_3^{18}} = \frac{3\times\frac{a}{8}}{\frac{21}{8}} = \frac{\frac{3a}{8}}{\frac{21}{8}} = \frac{3a}{21} = \frac{a}{7}$

۹-

$\log_4^3 = 0.8$

$\log_{12}^4 = \frac{\log_4^4}{\log_4^{12}} = \frac{\log_4^4}{\log_4^{4\times3}} = \frac{1}{\log_4^4 + \log_4^3} = \frac{1}{1+0.8} = \frac{1}{1.8} = \frac{10}{18} = \frac{5}{9}$

$\log_4^2 = \frac{1}{2}\log_4^4 = \frac{1}{2} = 0.5$

دریا حسینی

۱۰-

$(a\log 2)x^2 + ax + b\log 2 = 0$

$\xrightarrow{x=-1} (a\log 2)(-1)^2 + a(-1) + b\log 2 = 0 \longrightarrow$

$a\log 2 - a + b\log 2 = 0 \longrightarrow a(\log 2 - 1) + b\log 2 = 0$

$\log 2 - 1 = -(1 - \log 2) = -\log 5$ ($\log 5 = 1 - \log 2$)

$\log 5 = \log \frac{10}{2} = \log 10 - \log 2 = 1 - \log 2$

$a(-\log 5) + b\log 2 = 0 \longrightarrow -a(\log 5) + b(\log 2) = 0 \longrightarrow$

$b(\log 2) = a(\log 5) \longrightarrow \frac{b}{a} = \frac{\log_{10} 5}{\log_{10} 2} = \log_2 5$

$\sqrt{2}^{\frac{b}{a}} = \sqrt{2}^{\log_2 5} = 5^{\log_2 \sqrt{2}} = 5^{\frac{1}{2}} = \sqrt{5}$

$\log_2 \sqrt{2} = \log_2 2^{\frac{1}{2}} = \frac{1}{2}\log_2 2 = \frac{1}{2}$