1. $f(x)=3^{Ax+B}$ $\quad y=x^2 \to x=1 \to 3^{A+B}=1 \to A+B=0$

$\quad x=3 \to 3^{3A+B}=9 \to 3A+B=2$

$A=1 \quad B=-1$

$3^{Ax+B} \to 3^{x-1}$ نقطه تلاقی با y، $x=0$: $\quad 3^{-1}=\frac{1}{3}$

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2. $\log_2 (4^x+15)=x+3 \qquad 2^{(x+3)}=4^x+15$

$2^x \times 2^3=(2^x)^2+15$

$2^x=t \qquad t^2-8t+15=0$

$(t-5)(t-3)=0$

$t=3: \quad 2^x=3 \quad x=\log_2 3$

$t=5: \quad 2^x=5 \quad x=\log_2 5$

$\log_2 5+\log_2 3=\log_2 15$

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3. $(\log_{21} 3)^2+\log_{21} 147 \times \log_{21} 1323 \to \log_{21} 3\times(21)^2=2+\log_{21} 3$

$\log_{21} 21\times 7=\log_{21} 21+\log_{21} 7=1+\log_{21} 21-\log_{21} 3=2-\log_{21} 3$

$(\log_{21} 3)^2+(2-\log_{21} 3)+(2+\log_{21} 3)=(\log_{21} 3)^2+4-(\log_{21} 3)^2=4$

$4-(\log_{21} 3)^2$

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4. $\log (x^2-2x+1)+3\log (1-x)=5$

$x^2-2x+1=(x-1)^2=(1-x)^2 \to 2\log(1-x)+3\log(1-x)=5$

$5\log(1-x)=5 \to \log(1-x)=1 \qquad 1-x=10 \qquad x=-9$

$\log_3 (1-x) \to \log_3 9=2$

اعظم رضایی

5. $\log_2 (x^2+2x+4) + \log_2 (x-2) = 3 \to \log_2 (x^2+2x+4)(x-2) = 3$

$\log_2 (x^3-8) = 3 \to x^3 - 8 = 8 \to x^3 = 16 \quad x = 2^{\frac{4}{3}}$

$\log_{\sqrt[3]{2}} x \to \log_{2^{\frac{1}{3}}} 2^{4/3} \to 4\log_2 2 = 4$

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6. $\log (2-x) - \log \frac{1}{(x-2)^2} = 3$

$\log (2-x) - \log (2-x)^{-2} = 3$

$\to \log (2-x) + 2\log (2-x) = 3 \quad 3\log (2-x) = 3 \quad \log (2-x) = 1$

$x = -8$

$\log_{\sqrt{2}} -x = \log_{2^{\frac{1}{2}}} 2^3 = 6\log_2 2 = 6$

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7. $3^{(x^2-2)} = 81^x \to 3^{(x^2-2)} = 3^{4x} \quad x^2-2 = 4x$

$\to x^2 - 4x - 2 = 0 \to x = 2 \pm \sqrt{6}$

$\log_6 (x-2) \to x = 2+\sqrt{6} \to \log_6 (2+\sqrt{6}-2) = \log_6 \sqrt{6} = \frac{1}{2}$

$\to x = 2-\sqrt{6} \to \log_6 -\sqrt{6}$ ✗

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8. $\log_3 2 = \frac{5}{8} \quad \log_{18} 8 = \log_{18} 2^3 = 3\log_{18} 2 \to \frac{\log_3 2}{\log_3 18} =$

$\frac{\log_3 2}{\log_3 3^2 + \log_3 2} = \frac{\frac{5}{8}}{2+\frac{5}{8}} = \frac{\frac{5}{8}}{\frac{21}{8}} = \frac{5}{21}$

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9. $\log_4 3 = 0.8 \to \frac{1}{2}\log_2 3 = \frac{4}{5} \quad \log_2 3 = \frac{8}{5}$

$\log_{12} 6 = \frac{\log_2 6}{\log_2 12} = \frac{1+\log_2 3}{2+\log_2 3} = \frac{1+\frac{8}{5}}{2+\frac{8}{5}} = \frac{\frac{13}{5}}{\frac{18}{5}} = \frac{13}{18}$

$(a\log_2 r)x^2 + ax + b\log_2 r = 0 \xrightarrow{x=-1} (a\log_2 r) - a + b\log_2 r = 0$ .10

$b\log_2 r = a - a\log_2 r \Rightarrow b\log_2 r = a(1-\log_2 r)$

$\frac{b}{a} = \frac{1-\log_2 r}{\log_2 r} \longrightarrow \frac{\log_2 5}{\log_2 r} = \log_r^5$

$(\sqrt{5})^{\frac{b}{a}} = r^{\frac{1}{r}\times\log_r^5} = r^{\log_r^{(5)^{\frac{1}{r}}}} = r^{\log_r^{\sqrt{5}}} = \sqrt{5}^{\log_r^r} = \sqrt{5}$

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