پریا امینی

۲۰

۱-

$\log_n m = a \to m = n^a$

$\log_{mn} m^2 n = b \xrightarrow{m=n^a} \log_{n^a n} (n^a)^2 n = \log_{n^{a+1}} n^{2a+1} = b \to b = \frac{2a+1}{a+1} \cancel{\log_n n}^{1} = \frac{2a+1}{a+1}$

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$\frac{2a+1}{a+1} = \frac{2a+2}{a+1} - \frac{1}{a+1} = 2 - \frac{1}{a+1}$

$a>0 \to \frac{1}{a+1} < 1 \to 1 < \underbrace{2-\frac{1}{a+1}}_{b} < 2 \to [b]=1$

۲-

الف) $y=\sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \to \log_{\frac{1}{2}} x \to x>0$

$\hookrightarrow \log_{\frac{1}{2}} x \neq 0 \to x \neq 1$

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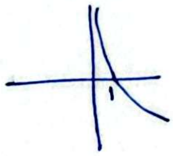
$\frac{x}{\log_{\frac{1}{2}} x} \geq 0 \xrightarrow{x \neq 0} \frac{x}{\log_{\frac{1}{2}} x} > 0 \xrightarrow{x>0} \log_{\frac{1}{2}} x > 0$



$\to$ به ازای $0<x<1$ عبارت مثبت است

اشتراک $\to 0<x<1 = D_y$

ب) $\frac{\log_4 (x^2-x-2)}{\underbrace{\sqrt{x^2-1}+1}_{\text{همواره غیر صفر و مثبت}}} \to x^2-x-2>0$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & 0 & - & 0 & + \end{array} \to \boxed{\begin{array}{l} x>2 \\ x<-1 \end{array}}$

$\sqrt{x^2-1} \to x^2-1 \geq 0 \to (x-1)(x+1) \geq 0$ $\begin{array}{ccccc} & -1 & & 1 & \\ + & 0 & - & 0 & + \end{array} \to \begin{array}{l} x \geq 1 \\ x \leq -1 \end{array}$

اشتراک $\to x>2 \cup x<-1 \to (-\infty,-1) \cup (2,+\infty) = D_y$

۳-

۵

$2\log_9 a + \log_a \sqrt{9} = 2 \to 2\log_9 a + \log_a 3 = 2 \to 2\log_{3^2} a + \log_a 3 = 2 \to \underbrace{2\times\frac{1}{2}}_{1} \times \log_3 a + \log_a 3 = 2$

$\log_3 a + \log_a 3 = 2 \to \frac{1}{\log_a 3} + \log_a 3 = 2 \to \frac{1}{t} + t = 2 \xrightarrow{\times t} 1+t^2-2t=0 \to (t-1)^2=0 \to t=1$

$\log_a 3 = 1 \to a=3$

پریا امینی

-۴

$\log 2 \simeq 0{,}3$

$\log 3 \simeq 0{,}4$

$\log 5 = \log \frac{10}{2} = \cancel{\log 10}^{\,1} - \cancel{\log 2}^{\,0{,}3} = 0{,}7$

$\log \frac{5}{3} = \log 5 - \log 3 = 0{,}7 - 0{,}4 = 0{,}3$

$\log 9 = \log 3\times 3 = \log 3^2 = 2\log 3 = 2\times 0{,}4 = 0{,}8$

$\log 15 = \log 5 + \log 3 = 0{,}7 + 0{,}4 = 1{,}1$

$(\log \frac{5}{3})x^2 + (\log 9)x - \log 15 = 0 \longrightarrow 0{,}3x^2 + 0{,}8x - 1{,}1 = 0 \xrightarrow{\times 10} 3x^2 + 8x - 11 = 0 \longrightarrow$

جمع ضرایب $= 0 \longrightarrow x = 1 \ , \ x = \frac{-11}{3}$ $\xrightarrow{\text{اختلاف ریشه‌ها}}$ $|1-(-\frac{11}{3})| = \boxed{\frac{14}{3}}$

-۵

$\log_2 7 = 2{,}8$

$\log_5 2 = 0{,}5$

$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{3}{3{,}8} \simeq 0{,}78$

$\log_2 14 = \cancel{\log_2 7}^{\,2{,}8} + \cancel{\log_2 2}^{\,1} = 3{,}8$

$\log_2 10 = \cancel{\log_2 2}^{\,1} + \cancel{\log_2 5}^{\,2} = 3$

$\log_2 5 = \frac{1}{\log_5 2} = \frac{1}{\frac{1}{2}} = 2$

-۶

$\log_3 5 = 1{,}5 \qquad \log_2 3 = 1{,}6 \longrightarrow \log_3 2 = \frac{1}{\log_2 3} = \frac{1}{\frac{16}{10}} = \frac{10}{16}$

$\log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{\log_3 2 + \cancel{\log_3 3}^{\,1}}{\cancel{\log_3 3}_{\,1} + \log_3 5} = \frac{\frac{10}{16}+1}{1+1{,}5} = \frac{1{,}625}{2{,}5} = 0{,}65$

پریا امینی

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$\log_{۸}^{۱۸} = m \rightarrow \dfrac{\log_{۲}^{۱۸}}{\log_{۲}^{۸}} = \dfrac{\log_{۲}^{۲}+\log_{۲}^{۹}}{\log_{۲}^{۲^۳} \rightarrow ۳} = \dfrac{۱+\log_{۲}^{۳^۲}}{۳} = \dfrac{۱+۲\log_{۲}^{۳}}{۳} = m \rightarrow ۱+۲\log_{۲}^{۳} = ۳m \rightarrow \log_{۲}^{۳} = \dfrac{۳m-۱}{۲}$

$\log_{۴}^{۱۲} = ?$

$\rightarrow \dfrac{\log_{۲}^{۱۲}}{\log_{۲}^{۴}} = \dfrac{\log_{۲}^{۳}+\log_{۲}^{۲^۲}}{\log_{۲}^{۲^۲}} = \dfrac{\log_{۲}^{۳}+۲}{۲} \rightarrow \dfrac{\dfrac{۳m-۱}{۲}+\dfrac{۴}{۲}}{۲} = \dfrac{\dfrac{۳m+۳}{۲}}{۲} = \boxed{\dfrac{۳m+۳}{۴}}$

۲

۸-

$(۰/۴)^{۲x-۱} = \left(\dfrac{۱۲۵}{۸}\right)^{x^۲} \rightarrow \left(\dfrac{۴}{۱۰}\right)^{۲x-۱} = \left(\dfrac{۵^۳}{۲^۳}\right)^{x^۲} \rightarrow \left(\dfrac{۲}{۵}\right)^{۲x-۱} = \left(\dfrac{۵}{۲}\right)^{۳x^۲} \rightarrow$

$\left(\left(\dfrac{۵}{۲}\right)^{-۱}\right)^{۲x-۱} = \left(\dfrac{۵}{۲}\right)^{۳x^۲} \rightarrow \cancel{\left(\dfrac{۵}{۲}\right)}^{-۲x+۱} = \cancel{\left(\dfrac{۵}{۲}\right)}^{۳x^۲} \rightarrow -۲x+۱ = ۳x^۲ \rightarrow ۳x^۲+۲x-۱=۰$

۲

جمع ضرایب اولی و آخری = وسطی $\rightarrow x=-۱ ,\ x=\dfrac{۱}{۳}$

$\log_{۸}^{۹x+۱} = \log_{۸}^{۹(\frac{۱}{۳})+۱} = \log_{۸}^{۴} = \log_{۲^۳}^{۲^۲} = \dfrac{۲}{۳}\cancel{\log_{۲}^{۲}}^{\,۱} = \dfrac{۲}{۳}$

۹-

$\log_{۲}^{۳} = a$

$\log_{۸}^{b} = \dfrac{۲}{۳}(۱+a) \rightarrow \log_{۲^۳}^{b} = \dfrac{۲}{۳}(۱+a) \rightarrow \dfrac{۱}{۳}\log_{۲}^{b} = \dfrac{۲}{۳}(۱+a) \rightarrow \overbrace{۲\times\dfrac{۱}{۳}}^{\frac{۲}{۳}}\times\log_{۲}^{\sqrt{b}} = \cancel{\dfrac{۲}{۳}}(۱+a)$ ، $b=(\sqrt{b})^۲$

$\rightarrow \log_{۲}^{\sqrt{b}} = ۱+a \rightarrow \log_{۲}^{\sqrt{b}} = ۱+\log_{۲}^{۳} \rightarrow \log_{۲}^{\sqrt{b}} - \log_{۲}^{۳} = ۱ \rightarrow \log_{۲}^{\frac{\sqrt{b}}{۳}} = ۱ \rightarrow \dfrac{\sqrt{b}}{۳} = ۲ \rightarrow$

$\sqrt{b} = ۶ \rightarrow b = ۳۶$

$\log(۳b-۸) = \log(۳(۳۶)-۸) = \log(۱۰۸-۸) = \log ۱۰۰ = \boxed{۲}$

۲

پریا امینی

۱۰ -

$-4ax^2+bx+\frac{1}{2}c=0$

$S=\frac{-b}{-4a}=\frac{b}{4a}\xrightarrow{\text{عکس}}\frac{4a}{b}=\log 4$

$a=\frac{b+c}{2}\to 2a=b+c \longleftarrow b=\frac{4a}{\log_{10}4}$

$2a=\frac{4a}{\log_{10}4}+c\to c=2a-\frac{4a}{\log_{10}4}\to c=a\left(2-\frac{4}{\log 4}\right)\to\frac{c}{a}=2-\frac{4}{\log 4}=2-\frac{4}{2\log 2}$

$\to\frac{c}{a}=2-\frac{2}{\log 2}$

$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}=\left((2)^{\frac{-1}{8}}\right)^{2-\frac{2}{\log 2}}=2^{(-\frac{1}{8})\times(2-\frac{2}{\log 2})}=2^{(\frac{-2}{8}+\frac{2}{-8\log 2})}=2^{(-\frac{1}{4}+\frac{1}{4\log 2})}=$

$\hookrightarrow\frac{1}{\sqrt[8]{2}}=\frac{1}{(2)^{\frac{1}{8}}}=(2)^{-\frac{1}{8}}$

$2^{\frac{1}{4}(\frac{1}{\log 2}-1)}=2^{\frac{1}{4}(\log_2 10-1)}=\sqrt[4]{2^{\log_2 10}-1}=\sqrt[4]{\frac{2^{\log_2 10}}{2}}=\sqrt[4]{\frac{10^{\cancel{\log_2 2}\,1}}{2}}=\sqrt[4]{\frac{10}{2}}=\sqrt[4]{5}$

$\hookrightarrow\frac{1}{\log_{10}2}=\log_2 10$