آرشا یاوری — یازدهم دفتر ۸ — تکلیف شماره ۲۳

۱- $\log_n^m = a \rightarrow \dfrac{\log m}{\log n} = a \qquad b = \log_{mn}^{m^2 n} = \dfrac{2\log m + \log n}{\log m + \log n} = \dfrac{2a+1}{a+1}$

$\rightarrow \dfrac{2a+1}{a+1} = 1 + \dfrac{a}{a+1} \rightarrow 0 < \dfrac{a}{a+1} < 1 \rightarrow 1 < \underbrace{1 + \dfrac{a}{a+1}}_{b} < 2$

$\Rightarrow [b] = 1$

۲- الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}}^{x}}} \qquad \log_{\frac{1}{2}}^{x} \neq 0 \rightarrow x \neq 1$ ① $\qquad x > 0$ ②

$\dfrac{x}{\log_{\frac{1}{2}}^{x}} \geq 0 \xrightarrow{x>0} \log_{\frac{1}{2}}^{x} > 0 \rightarrow x < 1$ ③

$\Rightarrow D = ① \cap ② \cap ③ = (0,1)$

ب) $y = \dfrac{\log_{e}^{(x^2-x-2)}}{\sqrt{x^2-1}+1} \qquad x^2-x-2 = (x-2)(x+1) > 0$

	-1		2	
$+$	0	$-$	0	$+$

$\rightarrow (-\infty,-1) \cup (2,+\infty)$ ①

$x^2-1 \geq 0 \rightarrow (x+1)(x-1) \geq 0$

	-1		$+1$	
$+$	0	$-$	0	$+$

$\rightarrow (-\infty,-1] \cup [1,+\infty)$ ②

$\sqrt{x^2-1}+1 \neq 0 \rightarrow$ همیشه برقرار

$D = ① \cap ② = (-\infty,-1) \cup (2,+\infty)$

۳- $2\log_9^a + \log_a^3 = 2 \rightarrow \log_3^a + \dfrac{1}{\log_3^a} = 2$

$\rightarrow \log_3^a = y \rightarrow y + \dfrac{1}{y} = 2 \rightarrow y^2 - 2y + 1 = 0 \rightarrow y = 1 \rightarrow \log_3^a = 1$

$\rightarrow 3^1 = a \rightarrow a = 3$

Subject. Day. Month. Year.

$\log_{10}\frac{10}{2}$

$\log_{10}\frac{5}{3}=\log_{10}5-\log_{10}3=\log_{10}10-\log_{10}2-\log_{10}3=1-0/3-0/4=0/3$ ـ۴

$\log 9=2\log 3=2(0/4)=0/8$

$\log 15=\log 3+\log 5=\log 3+\log 10-\log 2=0/4+1-0/3=1/1$

$\Rightarrow (0/3)x^2+(0/8)x-1/1=0 \rightarrow x=1\ ,\ x=\frac{-11}{3} \qquad 1-\left(\frac{-11}{3}\right)=\frac{14}{3}$

$\log_{14}10=\frac{\log_2 10}{\log_2 14}=\frac{\log_2 5+\log_2 2}{\log_2 2+\log_2 7}=\frac{\frac{10}{5}+1}{1+\frac{28}{10}}=\frac{30}{38}=\frac{15}{19}$ ـ۵

$\log_{15}6=\frac{\log_3 6}{\log_3 15}=\frac{\log_3 2+\log_3 3}{\log_3 5+\log_3 3}=\frac{\frac{10}{16}+1}{\frac{15}{10}+1}=\frac{26}{16}\times\frac{10}{25}=0/65$ ـ۶

$\log_8 18=\frac{\log_2 18}{\log_2 8}=\frac{\log_2 18}{3}=m \rightarrow \log_2 18=3m \rightarrow \log_2 18=\overset{1}{\cancel{\log_2 2}}+2\log_2 3$ ـ۷

$\rightarrow \log_2 3=\frac{3m-1}{2} \qquad \log_4 12=\frac{\log_2 12}{\log_2 4}=\frac{\log_2 12}{2} \rightarrow \log_2 12=\overset{2}{\cancel{\log_2 4}}+\log_2 3$

$\rightarrow \log_2 12=2+\frac{3m-1}{2}=\frac{3m+3}{2} \rightarrow \log_4 12=\frac{\frac{3m+3}{2}}{2}=\frac{3m+3}{4}=\frac{3(m+1)}{4}$

$\frac{125}{8}=\left(\frac{5}{2}\right)^3 \qquad 0/4=\frac{2}{5}=\left(\frac{5}{2}\right)^{-1}$ ـ۸

$\Rightarrow \left[\left(\frac{5}{2}\right)^3\right]^{x^2}=\left(\frac{5}{2}\right)^{3x^2}=\left(\frac{5}{2}\right)^{-(2x-1)}=\left(\frac{5}{2}\right)^{-2x+1}$

$\Rightarrow 3x^2=-2x+1 \rightarrow 3x^2+2x-1=0 \rightarrow x=-1\ ,\ x=\frac{+1}{3}$

$9x+1>0 \rightarrow 9x>-1 \rightarrow x>\frac{-1}{9} \Rightarrow \underset{\text{غ ق ق}}{x=-1}\ ,\ \boxed{x=\frac{1}{3}}$ ✓

$\Rightarrow \log_8 4=\log_{2^3}2^2=\frac{2}{3}$

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$$\log_{۸} b = \frac{\log_{۲} b}{\log_{۲} ۸} = \frac{\log_{۲} b}{۳} = \frac{۲}{۳}(۱+a) \xrightarrow{\times ۳} \log_{۲} b = ۲+۲a$$ ۹

$$b = ۲^{(۲+۲a)} = ۲^{۲} \times ۲^{۲a} \qquad \log_{۲} ۳ = a \longrightarrow ۲^{a} = ۳ \longrightarrow ۲^{۲a} = ۳^{۲} = ۹$$

$$\Rightarrow b = ۴ \times ۹ = ۳۶ \longrightarrow ۳b - ۸ = ۳(۳۶) - ۸ = ۱۰۸ - ۸ = ۱۰۰$$

$$\rightarrow \log_{۱۰} ۱۰۰ = ۲$$

$$S = \frac{b}{۴a} \rightarrow \frac{۱}{S} = \log ۴ \rightarrow \frac{۴a}{b} = \log ۴ = ۲\log ۲ \rightarrow \log ۲ = \frac{۲a}{b}$$ ۱۰

$$a = \frac{b+c}{۲} \rightarrow ۲a = b+c \rightarrow c = ۲a - b \xrightarrow{\div a} \frac{c}{a} = ۲ - \frac{b}{a} \quad , \quad \frac{b}{a} = \frac{۴}{\log ۴}$$

$$\rightarrow \frac{c}{a} = ۸$$

$$\rightarrow \left(\frac{۱}{\sqrt[۸]{۲}}\right)^{\frac{c}{a}} \rightarrow \frac{۱}{\sqrt[۸]{۲}} = ۲^{\frac{-۱}{۸}} \rightarrow ۲^{\frac{-c}{۸a}} \Rightarrow ۲^{\frac{-۸}{۸}} = ۲^{-۱} = \frac{۱}{۲}$$