(۱

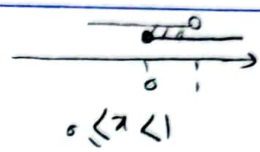
$$\log_n m = a \quad , \quad \log_{mn} m^2 n = b \qquad a>0$$

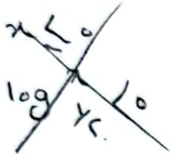
$$\log_n m = \frac{\log_{mn} m}{\log_{mn} n} = \frac{\frac{1}{\log_m mn}}{\frac{1}{\log_n mn}} = \frac{\log_n n + \log_n m}{\log_m m + \log_m n} = \frac{1+a}{1+\frac{1}{a}} \rightarrow \log_{mn} m^2 + \log_{mn} n = 2\log_{mn} m + \log_{mn} n = 2(1+a) + 1 + \frac{1}{a} =$$

$$2 + 2a + 1 + \frac{1}{a} = 3 + (2a + \frac{1}{a}) \rightarrow [b] = [3 + (2a + \frac{1}{a})] = 3 + [2a + \frac{1}{a}]$$

(۲

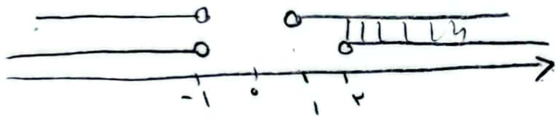
الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}}$ $\qquad \dfrac{x}{\log_{\frac{1}{2}} x} \geq 0$ $\qquad x \geq 0$, $\log_{\frac{1}{2}} x > 0 \rightarrow x < 1$ $\qquad 0 < x < 1$





ب) $y = \dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1}$

$$\rightarrow \begin{cases} 1)\ x^2 - x - 2 > 0 \rightarrow (x-2)(x+1) = 0 \rightarrow x=2,\ x=-1 \quad (-\infty,-1)\cup(2,+\infty) \\ 2)\ x^2 - 1 > 0 \rightarrow x^2 > 1 \rightarrow |x| > 1 \rightarrow \{x > 1 \text{ یا } x < -1 \end{cases}$$



$$\rightarrow D = (-\infty,-1)\cup(2,+\infty)$$

(۳

$$2\log_x a + \log_a \sqrt{x} = 2 \qquad x = 9$$

$$\frac{2}{\log_a x} + \frac{1}{2}\log_a x = 2 \xrightarrow{x=9} \frac{2}{2\log_a 3} + \log_a 3 = 2 \xrightarrow{\log_a 3 = x} \frac{1}{x} + x = 2$$

$$x^2 - 2x + 1 = 0 \rightarrow (x-1)^2 = 0 \rightarrow x = 1 \rightarrow \log_a 3 = 1 \rightarrow a = 3$$

(۴

$$\log 2 \simeq 0/3 \quad , \quad \log 3 \simeq 0/4 \qquad \rightarrow (\log \frac{5}{3})x^2 + (\log 9)x - \log 15 = 0$$

$$\log 5 = 1 - \log 2 = 1 - 0/3 = 0/7 \qquad \log 9 = \log 3^2 = 2\log 3 = 2 \times 0/4 = 0/8$$

$$\log 15 = \log 5 \times 3 = \log 5 + \log 3 = 1/1 \quad , \quad \log \frac{5}{3} = \log 5 - \log 3 = 0/3$$

$$\rightarrow (\log \frac{5}{3})x^2 + (\log 9)x - \log 15 = 0 \rightarrow 0/3x^2 + 0/8x - 1/1 = 0 \xrightarrow{a+b+c=0}$$

$$\begin{cases} x_1 = 1 \\ x_2 = \frac{-1/1}{0/3} \end{cases} \rightarrow x_1 - x_2 = \frac{14}{3}$$

(۵

$$\log_2 7 = 2/8 \quad , \quad \log_5 2 = 0/5 \quad \rightarrow \log_{14} 10$$

$$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 2 + \log_2 5}{\log_2 2 + \log_2 7} = \frac{1+2}{1+2/8} = \frac{3}{3/8}$$

۶-

$\log_{۲}^{۵} = ۱/۵$ ، $\log_{۲}^{۳} = ۱/۶$ $\log_{۱۵}^{۶}$

$$\log_{۱۵}^{۶} = \frac{\log_{۳}^{۶}}{\log_{۳}^{۱۵}} = \frac{\log_{۳}^{۲}+\log_{۳}^{۳}}{\log_{۳}^{۵}+\log_{۳}^{۳}} = \frac{\frac{۱}{۱/۶}+۱}{۱/۵+۱} = \frac{\frac{۱۳}{۸}}{\frac{۵}{۲}} = \frac{۱۳}{۲۰} = ۰/۶۵$$

۷-

$\log_{۸}^{۱۸} = m \rightarrow \log_{۴}^{۱۲} = ?$

$$\log_{۸}^{۱۸} = \log_{۸}^{۲\times۳^۲} = \log_{۸}^{۲} + ۲\log_{۸}^{۳} = \log_{۲^۳}^{۲} + ۲\log_{۸}^{۳} = m \rightarrow \frac{۱}{۳}\log_{۲}^{۲} + ۲\log_{۸}^{۳} = m$$

$$\rightarrow ۲\log_{۸}^{۳} = m - \frac{۱}{۳} \rightarrow \log_{۸}^{۳} = \frac{m}{۲} - \frac{۱}{۶}$$

$$\log_{۴}^{۱۲} = \frac{\log_{۸}^{۱۲}}{\log_{۸}^{۴}} = \frac{\log_{۲^۳}^{۲^۲}+\log_{۸}^{۳}}{\log_{۲^۲}^{۲^۳}} = \frac{\frac{۲}{۳}+\frac{m}{۲}-\frac{۱}{۶}}{\frac{۳}{۲}} = \frac{\frac{۱}{۲}+\frac{m}{۲}}{\frac{۳}{۲}} = \frac{\frac{(m+۱)}{۲}}{\frac{۳}{۲}} = \frac{(m+۱)}{۳}$$

۸-

$$(۰/۴)^{۲x-۱} = \left(\frac{۱۲۵}{۸}\right)^{x^۲} \rightarrow \left(\frac{۲}{۵}\right)^{۲x-۱} = \left(\frac{۵}{۲}\right)^{۳x^۲} \rightarrow \left(\frac{۵}{۲}\right)^{۱-۲x} = \left(\frac{۵}{۲}\right)^{۳x^۲}$$

$$\rightarrow ۳x^۲+۲x-۱=۰ \xrightarrow{b=a+c} \begin{cases} x=-۱ \\ x=\frac{۱}{۳} \end{cases} \rightarrow \log_{۸}(۹x+۱) \xrightarrow{x=\frac{۱}{۳}} \log_{۸}^{۴} = \frac{۲}{۳}$$

۹-

$\log_{۲}^{۳} = a$ ، $\log_{۸}^{b} = \frac{۲}{۳}(۱+a)$

$$\log_{۸}^{b} = \log_{۲^۳}^{b} = \frac{۲}{۳}(۱+a) \rightarrow \frac{۱}{۳}\log_{۲}^{b} = \frac{۲}{۳}(۱+a) \rightarrow \log_{۲}^{b} = ۲ + \log_{۲}^{۳}$$

$$\log_{۲}^{b} - \log_{۲}^{۳} = ۲ \rightarrow \log_{۲}^{\frac{b}{۳}} = ۲ \rightarrow \frac{b}{۳} = ۴ \rightarrow b = ۱۲$$

$$\log(۳^{x+۲} - ۷) = \log^{۲۸}$$

۱۰-

$$-۴ax^۲ + bx + \frac{۱}{۲}c = ۰$$

$$S = x_۱ + x_۲ \rightarrow \frac{۱}{S} = \frac{۱}{x_۱+x_۲} = ۲\log ۲ \rightarrow \frac{۱}{\frac{b}{۴a}} = \frac{۴a}{-b} = ۲\log ۲ \rightarrow$$

$$\frac{۲a-c}{۲a} = \log_{۲}^{۱۰} \rightarrow ۱ - \frac{c}{۲a} = \log_{۲}^{۱۰} \rightarrow \frac{c}{۲a} = ۱ - \log_{۲}^{۱۰} \xrightarrow{\times ۲} \frac{c}{a} = ۲ - \log_{۲}^{۱۰۰}$$

$$\left(\frac{۱}{\sqrt[۸]{۲}}\right)^{\frac{c}{a}} = \left(\frac{۱}{۲^{\frac{۱}{۸}}}\right)^{۲-\log_{۲}^{۱۰۰}} = \left(۲^{-۱}\right)^{۲-\log_{۲}^{۱۰۰}} \rightarrow \left(\frac{۱}{۱۰۰}\right)^{\frac{۱}{۷}} = \left(\frac{۲}{۵}\right)^{۷}$$

$$\left(۵^{۲}\right)^{\frac{۱}{۷}} = ۵^{\frac{۲}{۷}} = \sqrt[۷]{۵^۲}$$