دبیرستان ..... یازدهم دفتر A تکلیف شماره ۲۳ ۱۷,۲۵

۱)

$\log_n m = a \quad , \log_{mn} m^2 n = b \quad a>0 \quad \log_n m = \frac{\log_{mn} m}{\log_{mn} n} = \frac{\frac{1}{\log_m mn}}{\frac{1}{\log_n mn}} =$ ۱

$\frac{\log_n m + \log_n n}{\log_m m + \log_m n} = \frac{1+a}{1+\frac{1}{a}} \to \log_{mn} m^2 + \log_{mn} n = 2\log_{mn} m + \log_{mn} n = 2(1+a) + 1 + \frac{1}{a} =$

$2 + 2a + 1 + \frac{1}{a} = 3 + (2a + \frac{1}{a}) \to [b] = [3 + (2a + \frac{1}{a})] = 3 + [2a + \frac{1}{a}]$

۲)

الف) $y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \quad \frac{x}{\log_{\frac{1}{2}} x} \geq 0 \quad x > 0 \quad \log_{1/2} x > 0 \to x < 1 \quad 0 < x < 1$

۲

$x < 0$, $\log_{1/2} > 0$

ب) $y = \frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \to$ ۱) $x^2 - x - 2 > 0 \to (x-2)(x+1) = 0 \to x = 2, x = -1$ $\begin{array}{ccc} -1 & & 2 \\ + & - & + \end{array}$ ۱,۷۵

$(-\infty, -1) \cup (2, +\infty)$

۲) $x^2 - 1 > 0 \to x^2 > 1 \to |x| > 1 \to \{ x > 1$ یا $x < -1$

$\to D = (-\infty, -1) \cup (2, +\infty)$

-۱ ۰ ۱ ۲

۳)

$2\log_x a + \log_a \sqrt{x} = 2 \qquad x = 9$ ۲

$\frac{2}{\log_a x} + \frac{1}{2}\log_a x = 2 \xrightarrow{x=9} \frac{\cancel{2}}{\cancel{2}\log_a 3} + \log_a 3 = 2 \xrightarrow{\log_a 3 = x} \frac{1}{x} + x = 2$

$x^2 - 2x + 1 = 0 \to (x-1)^2 = 0 \to x = 1 \to \log_a 3 = 1 \to a = 3$

۴)

$\log 2 \simeq 0/3 \quad , \log 3 \simeq 0/4 \quad \to (\log \frac{5}{3}) x^2 + (\log 9) x - \log 15 = 0$

$\log 5 = 1 - \log 2 = 1 - 0/3 = 0/7 \qquad \log 9 = \log 3^2 = 2\log 3 = 2 \times 0/4 = 0/8$

$\log 15 = \log 5 \times 3 = \log 5 + \log 3 = 1/1 \quad , \log \frac{5}{3} = \log 5 - \log 3 = 0/3$ ۲

$\to (\log \frac{5}{3}) x^2 + (\log 9) x - \log 15 = 0 \to 0/3 x^2 + 0/8 x - 1/1 = 0 \xrightarrow{a+b+c=0}$

$\begin{cases} x_1 = 1 \\ x_2 = \frac{-1/1}{0/3} \end{cases} \to x_1 - x_2 = \frac{14}{3}$

۵)

$\log_2 7 = 2/8 \quad , \log_5 2 = 0/5 \to \log_{14} 10$

$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\cancel{\log_2 2} + \log_2 5}{\cancel{\log_2 2} + \log_2 7} = \frac{1+2}{1+2/8} = \frac{3}{3/8}$ ۲

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6-

$\log_3 5 = 1/5 \,,\; \log_2 3 = 1/6 \qquad \log_{15} 6$

$\log_{15} 6 = \dfrac{\log_3 6}{\log_3 15} = \dfrac{\log_3 2 + \log_3 3}{\log_3 5 + \log_3 3} = \dfrac{\frac{1}{1/6}+1}{1/5+1} = \dfrac{\frac{13}{8}}{\frac{5}{2}} = \dfrac{13}{20} = 0/65$

1

7-

$\log_8 18 = m \longrightarrow \log_4 12 = ?$

$\log_8 18 = \log_8 2\times3^2 = \log_8 2 + 2\log_8 3 = \log_{2^3} 2 + 2\log_8 3 = m \rightarrow \frac{1}{3}\cancel{\log_2 2} + 2\log_8 3 = m$

$\rightarrow 2\log_8 3 = m - \frac{1}{3} \rightarrow \log_8 3 = \frac{m}{2} - \frac{1}{6}$

1

$\log_4 12 = \dfrac{\log_8 12}{\log_8 4} = \dfrac{\log_{2^3} 2^2 + \log_8 3}{\log_{2^3} 2^2} = \dfrac{\frac{2\times2}{3\times2} + \frac{m}{2} - \frac{1}{6}}{\frac{2}{3}} = \dfrac{\frac{1}{2}+\frac{m}{2}}{\frac{2}{3}} = \dfrac{\frac{(m+1)}{2}}{\frac{2}{3}} = \dfrac{3(m+1)}{4}$

(8

$(0/4)^{2x-1} = (\frac{125}{8})^{x^2} \longrightarrow (\frac{2}{5})^{2x-1} = (\frac{5}{2})^{x^3} \longrightarrow (\frac{5}{2})^{1-2x} = (\frac{5}{2})^{x^3}$

1

$\longrightarrow x^3 + 2x - 1 = 0 \xrightarrow{b=a+c} \begin{cases} x=-1 \\ x=\frac{1}{3} \end{cases} \longrightarrow \log_8 (9x+1) \xrightarrow{x=\frac{1}{3}} \log_8 4 = \frac{2}{3}$

(9

$\log_2 3 = a \,,\; \log_8 b = \frac{2}{3}(1+a)$

$\log_8 b = \log_{2^3} b = \frac{2}{3}(1+a) \rightarrow \frac{1}{\cancel{3}}\log_2 b = \frac{2}{\cancel{3}}(1+a) \rightarrow \log_2 b = 2 + \log_2 3$

$\log_2 b - \log_2 3 = 2 \longrightarrow \log_2 \frac{b}{3} = 2 \rightarrow \frac{b}{3} = 4 \rightarrow b = 12$

0/5

$\log\,(3\times12-8) = \log 28$

(10

$-4ax^2 + bx + \frac{1}{2}c = 0$

$S = x_1 + x_2 \rightarrow \frac{1}{S} = \frac{1}{x_1+x_2} = 2\log 2 \rightarrow \dfrac{1}{\frac{b}{4a}} = \dfrac{4a}{\cancel{b}\;\, 2a-c} = \cancel{2}\log 2 \rightarrow$

1

$\dfrac{2a-c}{2a} = \log_2 10 \longrightarrow 1 - \frac{c}{2a} = \log_2 10 \rightarrow \frac{c}{2a} = 1 - \log_2 10 \xrightarrow{\times 2} \frac{c}{a} = 2 - \log_2 100$

$(\frac{1}{\sqrt[8]{2}})^{\frac{c}{a}} = (\frac{1}{\sqrt[8]{2}})^{2-\log_2 100} = (2^{\frac{1}{8}})^{-2+\log_2 100} \rightarrow \quad (\frac{100}{4})^{\frac{1}{8}} = (25)^{\frac{1}{8}}$

$(5^2)^{\frac{1}{8}} = 5^{1/4} = \sqrt[4]{5}$

$\log_n^m = a \to m = n^a \qquad \log_{mn}^{m^2 n} = b \to \log_{n^{2a}\times n}^{n^{2a}\times n} = b$ (1

$\to \frac{2a+1}{a+1} = b \qquad b = 2 - \frac{1}{a+1} \xrightarrow{0<a} [b] = [2^-] = 1$

$\log_8^b = \frac{2}{3}(\log_2^2 + \log_2^3) = \frac{1}{\cancel{3}}\log_2^b = \frac{2}{\cancel{3}}(\log_2^6)$ (9

$\to \log_2^b = \log_2^{6^2} \to b = 36 \qquad \log^{3b-8} = \log^{100} = 2$