(۵)

$$\log_{۱۴}^{۱۰} = \frac{\log_{۲}^{۱۰}}{\log_{۲}^{۱۴}} = \frac{\log_{۲}^{۵} + \log_{۲}^{۲}}{\log_{۲}^{۲} + \log_{۲}^{۷}} \xrightarrow[\log_{۲}^{۵} = \frac{۱۰}{۵}]{\log_{۵}^{۲} = ۰,۵} \log_{۱۴}^{۱۰} = \frac{۲+۱}{۱+۲,۸} = \frac{۳}{۳,۸}$$

$$= \frac{۱۵}{۱۹}$$

(۲)

(۶)

$$\log_{۱۵}^{۶} = \frac{\log_{۳}^{۶}}{\log_{۳}^{۱۵}} = \frac{\log_{۳}^{۳} + \log_{۳}^{۲}}{\log_{۳}^{۵} + \log_{۳}^{۳}} \xrightarrow[\log_{۳}^{۲} = \frac{۱۰}{۱۶}]{\log_{۲}^{۳} = ۱,۶} \log_{۱۵}^{۶} = \frac{۱ + \frac{۵}{۸}}{۱,۵ + ۱}$$

$$= \frac{\frac{۱۳}{۸}}{\frac{۲۵}{۱۰}} = \frac{۱۳}{۲۰}$$

(۲)

(۷)

$$\log_{۴}^{۱۲} = \log_{۴}^{۴} + \log_{۴}^{۳} = ۱ + \log_{۴}^{۳}$$

$$m = \log_{۸}^{۱۸} = \frac{\log_{۴}^{۱۸}}{\log_{۴}^{۸}} = \frac{\log_{۴}^{۲} + \log_{۴}^{۹}}{\log_{۴}^{۲} + \log_{۴}^{۴}} \Rightarrow m = \frac{\frac{۱}{۲} + ۲\log_{۴}^{۳}}{\frac{۱}{۲} + ۱} \longrightarrow$$

$$\frac{۳}{۲}m = \frac{۱}{۲} + ۲\log_{۴}^{۳} \longrightarrow \frac{۳m}{۲} - \frac{۱}{۲} = ۲\log_{۴}^{۳} \longrightarrow \log_{۴}^{۳} = \frac{۳m}{۴} - \frac{۱}{۴} \longrightarrow$$

$$\log_{۴}^{۱۲} = ۱ + \frac{۳m}{۴} - \frac{۱}{۴} = \frac{۳}{۴}(m+۱)$$

(۲)

(۸)

$$\left(\frac{۱۲۵}{۸}\right)^{x^۲} = \left(\frac{۵^۳}{۲^۳}\right)^{x^۲} = \left(\frac{۲}{۵}\right)^{-۳x^۲} = (۰,۴)^{-۳x^۲} \longrightarrow$$

(۲)

$$-۳x^۲ = ۲x - ۱ \longrightarrow ۳x^۲ + ۲x - ۱ = ۰ \longrightarrow x = -۱ \times,\ x = \frac{۱}{۳} \longrightarrow$$

$$\log_{۸}^{۹\times\frac{۱}{۳}+۱} = \log_{۸}^{۴} = \frac{۲}{۳}\log_{۲}^{۲} = \frac{۲}{۳}$$

(۹)

$$\log_{۸}^{b} = \frac{۲}{۳}(۱ + \log_{۲}^{۳}) = \frac{۲}{۳}(\log_{۲}^{۲} + \log_{۲}^{۳}) = \frac{۲}{۳}\log_{۲}^{۶} = \log_{۸}^{۳۶} \longrightarrow$$

(۲)

$$b = ۳۶ \longrightarrow \log_{۱۰}^{(۳\times۳۶)-۸} = \log_{۱۰}^{۱۰۰} = ۲$$

① $\log_n m = a$, $\log_{mn} m^2 n = b$, $a>0$, $[b]=?$

$m=n^a \longrightarrow \log_{mn} m^2 n = \log_{n^a\times n} n^{2a}\times n = \log_{n^{a+1}} n^{2a+1} \Longrightarrow \frac{2a+1}{a+1}=b$

$\longrightarrow b=\frac{a+1}{a+1}+\frac{a}{a+1}=1+\frac{a}{a+1} \xrightarrow{a>0} 0<\frac{a}{a+1}<1 \longrightarrow [b]=1$

②

الف) $y=\sqrt{\frac{x}{\log_{\frac{1}{2}} x}} \longrightarrow ① \frac{x}{\log_{\frac12} x} \geq 0 \longrightarrow$ 0 , 1 : ن ت + ت − ، ② $x>0$

$① \cap ② \longrightarrow D_y=(0,1)$

ب) $y=\frac{\log_2 (x^2-x-2)}{\sqrt{x^2-1}+1} \longrightarrow ① x^2-x-2>0 \longrightarrow$ −1 , 2 : + ت − ت +

② $x^2-1\geq 0 \longrightarrow x^2\geq 1 \longrightarrow x\geq 1 \cup x\leq -1$, ③ $\sqrt{x^2-1}+1 \longrightarrow$ هیچ وقت این [illegible]

$① \cap ② \longrightarrow D_y=(-\infty,-1)\cup(2,+\infty)$

③

$2\log_9 a+\log_a 3=2 \longrightarrow$

$\log_3 a+\log_a 3=2 \longrightarrow \log_3 a+\frac{1}{\log_3 a}-2=0 \xrightarrow{\log_3 a=t} t+\frac{1}{t}-2=0 \longrightarrow$

$t^2-2t+1=0 \longrightarrow t=1 \longrightarrow \log_3 a=1 \longrightarrow a=3$

④

$\log 9 = 2\log 3 \simeq 0.8$, $\log 15 = \log 3+\log 5 \longrightarrow \log 5=\log\frac{10}{2} \longrightarrow$

$\log\frac{10}{2}=\log 10-\log 2 \longrightarrow \log 5 = 1-0.3=0.7 \longrightarrow \log 15 = 0.4+0.7 \simeq 1.1$

$\log\frac{5}{3}=\log 5-\log 3 = 0.7-0.4 \simeq 0.3 \longrightarrow$

$0.3x^2+0.8x-1.1=0 \longrightarrow a+b+c=0 \longrightarrow x=\frac{-11}{3}$, $x=1 \longrightarrow$

اختلاف ریشه‌ها $\longrightarrow D=\frac{3}{3}-\left(-\frac{11}{3}\right)=\frac{14}{3}$.

۱۰

$\frac{4a}{b} = 2\log^2 \longrightarrow \frac{2a}{b} = \log^2 \longrightarrow b = \frac{2a}{\log^2}$, $\frac{b+c}{2} = a \longrightarrow c = 2a - b$

$\frac{c}{a} = \frac{2a-b}{a} = 2 - \frac{b}{a} = 2 - \left(\frac{2a}{\log^2} \times \frac{1}{a}\right) = 2 - \frac{2}{\log^2} = 2\left(1 - \frac{1}{\log_2^{10}}\right) = 2 - 2\log_2^{10}$

$= 2 - \log_2^{100} \longrightarrow \frac{1}{\sqrt[8]{2}} = \left(2^{-\frac{1}{8}}\right)^{2 - 2\log_2^{10}} = \left(2^{-\frac{1}{8}}\right)^{2(1-\log_2^{10})} \longrightarrow$

$2^{-\frac{1}{4}\log_2^{0.2}} = 2^{\log_{2^4}^{5}} \longrightarrow 5^{\log_2^2} = 5^{\frac{1}{4}\log_2^2} = \sqrt[4]{5}$