تکلیف ۲۳ ۲۰ احمد مارانی

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$\log_n m = a \to m = n^a$

$\log_{mn} m^2 n$ } $\log_{n^a n} (n^a)^2 n = \log_{n^{a+1}} n^{2a+1} = b \to b = \frac{2a+1}{a+1}$

$\frac{2a+1}{a+1} = \frac{2a+2}{a+1} - \frac{1}{a+1} = 2 - \frac{1}{a+1} \xrightarrow{a>0} \frac{1}{a+1} < 1$

$\to 1 < \underbrace{2 - \frac{1}{a+1}}_{} < 2$ ۲

$1 < b < 2 \Rightarrow [b] = 1$

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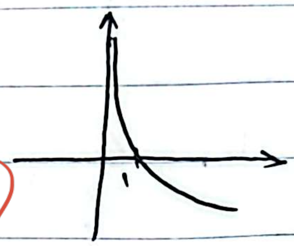
الف) $\sqrt{\frac{x}{\log_{1/2} x}} \Rightarrow x > 0,\ x \neq 1$

$\frac{x}{\log_{1/2} x} > 0 \xrightarrow{x>0} \log_{1/2} x > 0$ ۲



طبق نمودار داریم $\Rightarrow \ \to 0 < x < 1 = D$

ب) $\frac{\log(x^2-x-2)}{\sqrt{x^2-1}+1} \Rightarrow x^2-1 \geq 0 \to$ (-1: + − +, 1) $\quad x \geq 1,\ x \leq -1$ ①

$x^2-x-2 > 0 \to x < -1,\ x > 2$ ②

① ∩ ② $\Rightarrow D = (-\infty,-1) \cup (2,+\infty)$

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$2\log_x a + \log_a \sqrt{x} = 2$

↳ $2\log_9 a + \log_a \sqrt{9} = 2 \Rightarrow 2\log_{3^2} a + \log_a 3 = 2$

↳ $\log_3 a + \underbrace{\log_a 3}_{y} = 2$

$\frac{1}{y} + y = 2 \to y = 1 \to \log_a 3 = 1 \to \boxed{a=3}$ ۲

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$\log 2 \simeq 0.3 \,,\ \log 3 = 0.4$

$\left[\log \frac{5}{3}\right] x^2 + (\log 9)x - \log 15 = 0 \qquad |x_2 - x_1| = ?$

$\log 5 + \log 3 = \log 10 - \log 2 + \log 3 = 1.1$

$\log 3 + \log 3 = 0.8$

$\log 5 - \log 3 = \log 10 - \log 2 - \log 3 = 0.3$

$\Downarrow$

$0.3x^2 + 0.8x - 1.1 = 0 \rightarrow 3x^2 + 8x - 11$

$x^2 + 8x - 33 \rightarrow (x+11)(x-3) \rightarrow x_2 = -\frac{11}{3} \quad x_1 = 1$

$|x_2 - x_1| = \left|-\frac{11}{3} - 1\right| = \frac{14}{3}$

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$\log_2 7 = 2.8 \quad \log_5 2 = 0.5 \quad \log_{14} 10 = ?$

$\log_{14} 10 = \dfrac{\log_2 10}{\log_2 14}$

$\log_2 10 \rightarrow \log_2 5 + \log_2 2 = 3$

$\log_2 14 \rightarrow \log_2 7 + \log_2 2 = 3.8$

$\Rightarrow \dfrac{3}{3.8} \simeq 0.78$

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$\log_3 5 = 1.5 \quad \log_2 3 = 1.6 \quad \log_{15} 9 = ?$

$\log_{15} 9 = \dfrac{\log_3 9}{\log_3 15}$

$\log_3 9 \rightarrow \log_3 3 + \log_3 3 = \frac{10}{16} + 1$

$\log_3 15 \leftrightarrow \log_3 5 + \log_3 3 = 1 + 1.5$

$\Rightarrow \dfrac{1.625}{2.5} = 0.65$

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$\log_8 18 = m \quad \log_4 12 = ?$

$\log_8 18 = \dfrac{\log_2 18}{\log_2 8} = \dfrac{\log_2 2 + \log_2 9}{3} = \dfrac{1 + 2\log_2 3}{3} = m$

$\log_2 3 = \dfrac{3m-1}{2}$

$\log_4 12 = \dfrac{\log_2 12}{\log_2 4} = \dfrac{\log_2 4 + \log_2 3}{2}$

$= \dfrac{2 + \frac{3m-1}{2}}{2} = \dfrac{3m+3}{4}$

امحمد بارانی

۸- $(0/4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2}$ ، $\log_8^{(9x+1)}$ ؟ $=$

$$\left(\frac{5}{2}\right)^{1-2x} = \left(\frac{5}{2}\right)^{3x^2} \longrightarrow 1-2x = 3x^2 \rightarrow 3x^2+2x-1=0$$

$$\hookrightarrow x = -1 \text{ و } \boxed{\tfrac{1}{3} \text{ ق ق}}$$

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$$\tfrac{1}{3} \xrightarrow{\text{جایگذاری}} \log_8^4 = \log_{2^3}^{2^2} = \tfrac{2}{3}$$

۹- $\log_2^3 = a$ ، $\log_8^b = \frac{2}{3}(1+a)$ ، $\log(3b-8)$ ؟ $=$

$$\tfrac{1}{3}\log_2^b = \tfrac{2}{3}(1+a) \Rightarrow \cancel{\tfrac{2}{3}}\log_2^{\sqrt{b}} = \cancel{\tfrac{2}{3}}(1+a)$$

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$$\log_2^{\sqrt{b}} = a+1 \longrightarrow \log_2^{\sqrt{b}} = \log_2^3 + 1 \longrightarrow \log_2^{\sqrt{b}/3} = 1$$

$$\sqrt{b} = 6 \longrightarrow b = 36 \xrightarrow{\text{جایگذاری}} \log(100) = 2$$

۱۰- $-4ax^2 + bx + \frac{1}{2}c = 0$ ، $S = \frac{-b}{-4a} = \frac{b}{4a} \longrightarrow \frac{4a}{b} = \log 4$

$$b = \frac{4a}{\log_{10}^4} \longrightarrow a = \frac{b+c}{2} \rightarrow 2a = b+c$$

$$2a = \frac{4a}{\log 4} + c \rightarrow c = 2a - \frac{4a}{\log 4} \rightarrow \frac{c}{a} = 2 - \frac{4}{\log 4}$$

$$\longrightarrow \frac{c}{a} = 2 - \frac{2}{\log 2}$$

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$$\left(\frac{1}{\sqrt[8]{2}}\right)^{2-\frac{2}{\log 2}} \rightarrow \left(2^{-\frac{1}{8}}\right)^{2-\frac{2}{\log 2}} \longrightarrow 2^{\frac{1}{4}\left(\log_2^{10} - 1\right)}$$

$$\longrightarrow \sqrt[4]{2^{\log_2^{10} - 1}} = \sqrt[4]{\frac{2^{\log_2^{10}}}{2}} = \sqrt[4]{\frac{10^{\log_2^2}}{2}} = \boxed{\sqrt[4]{5}}$$