تکلیف شماره ۲۳ — آلاگل ساسانی

①

$\log_n m = a > 0$, $\log_{nm} m^2 n = b$, $[b] = ?$

$\log_{nm} m^2 n = \log_{mn} \cancel{mn} + \log_{mn} m \Rightarrow 1 + \frac{1}{\log_m m + \log_m n} = 1 + \frac{1}{1+\frac{1}{a}} = b \Rightarrow [b] = 1$

②

الف) $\sqrt{\frac{x}{\log_{\frac12} x}} \Rightarrow \frac{x}{\log_{\frac12} x} \geq 0$ — $\begin{array}{ccccc} & 0 & & 1 & \\ - & | & + & | & - \end{array}$ ③ $\Rightarrow ① \cap ② \cap ③ = (0, 1)$

$\log_{\frac12} x \neq 0 \Rightarrow x \neq 1$ ①

$x > 0$ ②

ب) $\frac{\log_2 (x^2 - x - 2)}{\sqrt{x^2-1} + 1} \Rightarrow$ $x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow$ $\begin{array}{ccccc} & -1 & & 2 & \\ + & | & - & | & + \end{array} \Rightarrow (-\infty, -1) \cup (2, +\infty)$ ①

$x^2 - 1 \geq 0 \Rightarrow 1 \leq x^2 \Rightarrow \begin{matrix} x \leq -1 \\ x \geq 1 \end{matrix} \Rightarrow (-\infty, -1] \cup [1, +\infty)$ ② $\quad D = (-\infty, -1) \cup (2, +\infty)$

$\sqrt{x^2-1} \neq -1 \Rightarrow$ همیشه برقرار

③

$2\log_x a + \log_a \sqrt{x} = 2 \Rightarrow 2\log_9 a + \log_a 3 = 2 \Rightarrow \log_3 a + \frac{1}{\log_3 a} = 2 \Rightarrow t + \frac{1}{t} = 2 \Rightarrow t^2 + 1 = 2t \Rightarrow$

$x = 9$ $\quad t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 \Rightarrow t = 1 \quad \log_3 a = 1 \Rightarrow a = 3$

④

$\log 3 = 0.48$, $\log 2 = 0.3$ $\quad x_2 - x_1$

$\left(\log \frac{5}{3}\right) x^2 + (\log 9) x - \log 15 = 0 \Rightarrow ((\log 10 - \log 2) - \log 3) x^2 + (2\log 3) x - (\log 3 + (\log 10 - \log 2)) = 0$

$(0.3) x^2 + (0.8) x \ (1.1) = 0 \Rightarrow 3x^2 + 8x \ 11 = 0 \Rightarrow x^2 + 8x \ 33$

$(x-3)(x+11) = \begin{matrix} x = -\frac{11}{3} \\ x = \frac{3}{3} = 1 \end{matrix} \quad -\frac{3}{3} - \frac{11}{3} = -\frac{14}{3} \Rightarrow \frac{14}{3}$

⑤

$\log_{14} 10 = \log_{14} 2 + \log_{14} 5 = \frac{1}{\log_2 14} + \frac{1}{\log_5 14} = \frac{1}{\underset{2.8}{\log_2 7} + \underset{1}{\cancel{\log_2 2}}} + \frac{1}{\log_5 7 + \underset{0.5}{\cancel{\log_5 2}}} \Rightarrow$ حاصل $\log_5 2 = 0.5$, $\log_2 7 = 2.8$

$\frac{1}{3.8} + \frac{1}{0.5 + \log_5 7} = \frac{1}{3.8} + \frac{2}{3.8} = \frac{3}{3.8}$

$\log_5 7 = \frac{\log_2 7}{\log_2 5} = \frac{2.8}{2} = 1.4$ $\quad \log_2 5 = \frac{1}{\log_5 2} = \frac{1}{0.5} = 2$

⑥

$\log_{15} 9 = \log_{15} 3 + \log_{15} 3 = \frac{1}{\underset{1.6}{\log_2 3} + \underset{2.4}{\cancel{\log_2 5}}} + \frac{1}{\underset{1.5}{\cancel{\log_3 5}} + \underset{1}{\cancel{\log_3 3}}} = \frac{1}{4} + \frac{1}{2.5} = \frac{6.5}{10} = 0.65$ $\quad \log_2 3 = 1.6 \quad \log_3 5 = 1.5$

$\log_2 5 = \frac{\log_3 5}{\log_3 2} = \frac{1.5}{\frac{1}{1.6}} = 2.4$

$$\log_{۸}^{۱۸} = m = \log_{۸}^{۲} + \log_{۸}^{۹} = \frac{۱}{۳}\cancel{\log_{۲}^{۲}} + \frac{۲}{۳}\log_{۲}^{۳} \Rightarrow m = \frac{۱}{۳} + \frac{۲}{۳}\log_{۲}^{۳} \Rightarrow \left(m - \frac{۱}{۳}\right) \times \frac{۳}{۲} = \log_{۲}^{۳} \ \star$$ ۷

$$\log_{۴}^{۱۲} = \log_{۴}^{۳} + \cancel{\log_{۴}^{۴}} = ۱ + \log_{۴}^{۳} = ۱ + \frac{۱}{۲}\log_{۲}^{۳} \overset{\star}{\Rightarrow} \log_{۴}^{۱۲} = ۱ + \frac{۱}{۲} \times \frac{۳}{۲}\left(m - \frac{۱}{۳}\right) \Rightarrow$$

$$\log_{۴}^{۱۲} = ۱ + \frac{۳}{۴}m - \frac{۱}{۴} = \log_{۴}^{۱۲} = \frac{۳}{۴} + \frac{۳}{۴}m = \frac{۳}{۴}(۱+m)$$

$$\left(\frac{۲}{۵}\right)^{۲x-۱} = \left(\frac{۵}{۲}\right)^{۳x^۲} \Rightarrow \left(\frac{۵}{۲}\right)^{۱-۲x} = \left(\frac{۵}{۲}\right)^{۳x^۲} \Rightarrow ۱ - ۲x = ۳x^۲ \Rightarrow ۳x^۲ + ۲x - ۱ = ۰ \Rightarrow x^۲ + ۲x - ۳ = ۰$$ ۸

$(x+۳)(x-۱) = ۰ \rightarrow x = \frac{-۳}{۳} = -۱$ غ ق ق

$\rightarrow x = \frac{۱}{۳}$ ق ق

$۹x + ۱ > ۰ \Rightarrow x > \frac{۱}{۹}$

$$\log_{۸}(۴) = \frac{۲}{۳}\log_{۲}^{۲} = \frac{۲}{۳}$$

$$\log_{۲}^{۳} = a \ , \ \log_{۸}^{b} = \frac{۲}{۳}(۱+a) \Rightarrow \log_{۸} b = \frac{۲}{۳}\left(\log_{۲}^{۲} + \log_{۲}^{۳}\right) \Rightarrow \frac{۲}{۳}\left(\log_{۲}^{۶}\right) \Rightarrow \log_{۸}^{۳۶} = \log_{۸} b$$ ۹

$$b = ۳۶ \Rightarrow \log(۳b - ۸) \xrightarrow{b=۳۶} \log ۱۰۰ = ۲$$

$$-۴ax^۲ + bx + \frac{۱}{۲}c = ۰ \longrightarrow \text{جمع ریشه‌ها}: \frac{-b}{-۴a} \Rightarrow \frac{۴a}{b} = \log ۴ \Rightarrow \frac{b}{a} = \log_{۴}^{۱۰^۴} = \log_{۲}^{۱۰^۲}$$ ۱۰

$$\frac{b+c}{۲} = a \ , \ ۲a = b + c \Rightarrow ۲ = \frac{b}{a} + \frac{c}{a} \Rightarrow \frac{c}{a} = ۲ - \log_{۲}^{۱۰۰} = ۲ - ۲\left(\log_{۲}^{۵} + ۱\right) = \frac{c}{a} = -۲\log_{۲}^{۵}$$

$$\left(\frac{۱}{\sqrt[۸]{۲}}\right)^{-۲\log_{۲}^{۵}} \Rightarrow \left(۲^{-\frac{۱}{۸}}\right)^{-۲\log_{۲}^{۵}} \Rightarrow ۲^{\frac{۱}{۴}\log_{۲}^{۵}} \Rightarrow ۲^{\log_{۲}^{\sqrt[۴]{۵}}} \Rightarrow \sqrt[۴]{۵}$$