تکلیف شماره ۲۳ ‏   ۲۵   لالا کرسانی

①

$\log_n^m = a > 0 \;,\; \log_{nm}^{m^2 n} = b \qquad [b] = ?$

$\log_{nm}^{m^2 n} = \log_{mn}^{mn} + \log_{mn}^{m} \Rightarrow 1 + \frac{1}{\log_m^m + \log_m^n} = 1 + \frac{1}{1+\frac{1}{a}} = b \Rightarrow [b] = 1$

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②

الف) $\sqrt{\frac{x}{\log_{\frac12} x}} \Rightarrow \frac{x}{\log_{\frac12}^x} \geq 0 \quad \begin{array}{c} 0 \quad\; 1 \\ - \;|\; + \;|\; - \end{array}$ ③ $\Rightarrow ① \cap ② \cap ③ \Rightarrow (0,1)$

$\log_{\frac12}^x \neq 0 \Rightarrow x \neq 1$ ①

$x > 0$ ②

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ب) $\frac{\log_2 (x^2-x-2)}{\sqrt{x^2-1}+1} \Rightarrow \quad x^2-x-2 > 0 \Rightarrow (x-2)(x+1) > 0 \Rightarrow \begin{array}{c} -1 \quad\; 2 \\ + \;|\; - \;|\; + \end{array} \Rightarrow (-\infty,-1)\cup(2,+\infty)$ ①

$x^2-1 \geq 0 \Rightarrow 1 \leq x^2 \Rightarrow \begin{array}{l} x\leq -1 \\ x \geq 1 \end{array} \Rightarrow (-\infty,-1]\cup[1,+\infty)$ ② $\qquad D = (-\infty,-1)\cup(2,+\infty)$

$\sqrt{x^2-1} \neq -1 \Rightarrow$ همیشه برقرار

③

$2\log_x^a + \log_a^{\sqrt{x}} = 2 \Rightarrow 2\log_9^a + \log_a^3 = 2 \Rightarrow \cancel{4}\log_3^a + \frac{1}{\log_3^a} = 2 \Rightarrow t + \frac{1}{t} = 2 \Rightarrow t^2 + 1 = 2t \Rightarrow$

$x = 9 \qquad t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 \Rightarrow t = 1 \qquad \log_3^a = 1 \Rightarrow a = 3$

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④

$\log 3 = 0.48 \;,\; \log 2 = 0.3 \qquad x_2 - x_1$

$\left[\log \frac{5}{3}\right] x^2 + (\log 9)x - \log 15 = 0 \Rightarrow ((\log_{10}^{10} - \log 2) - \log 3)x^2 + (2\log 3)x - (\log 3 + (\log 10 - \log 2)) = 0$

$(0.3)x^2 + (0.8)x \;\; (1.1) = 0 \Rightarrow 3x^2 + 8x \;\; 11 = 0 \Rightarrow x^2 + 8x \;\; 33$

$(x-3)(x+11) = \begin{cases} x = -\frac{11}{3} \\ x = \frac{3}{3} = 1 \end{cases} \qquad -\frac{3}{3} - \frac{11}{3} = -\frac{14}{3} \Rightarrow \frac{14}{3}$

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⑤

$\log_{14}^{10} = \log_{14}^{2} + \log_{14}^{5} = \frac{1}{\log_2^{14}} + \frac{1}{\log_5^{14}} = \frac{1}{\underbrace{\log_2^7}_{2.8} + \underbrace{\log_2^2}_{1}} + \frac{1}{\log_5^7 + \underbrace{\log_5^2}_{0.5}} \Rightarrow$ حاصل $\quad \log_5^2 = 0.5 \;,\; \log_2^7 = 2.8$

$\frac{1}{3.8} + \frac{1}{0.5 + \log_5^7} = \frac{1}{3.8} + \frac{2}{3.8} = \frac{3}{3.8}$

$\log_5^7 = \frac{\log_2^7}{\log_2^5} = \frac{2.8}{2} = 1.4 \qquad \log_2^5 = \frac{1}{\log_5^2} = \frac{1}{0.5} = 2$

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⑥

$\log_{15}^{9} = \log_{15}^{2} + \log_{15}^{3} = \frac{1}{\underbrace{\log_2^3}_{1.6} + \underbrace{\log_2^5}_{2.4}} + \frac{1}{\underbrace{\log_3^5}_{1.5} + \underbrace{\log_3^3}_{1}} = \frac{1}{4} + \frac{1}{2.5} = \frac{6.5}{10} = 0.65 \qquad \log_2^3 = 1.6 \qquad \log_3^5 = 1.5$

$\log_2^5 = \frac{\log_3^5}{\log_3^2} = \frac{1.5}{\frac{1}{1.6}} = 2.4$

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(7)

$\log_{8} 18 = m = \log_{8} 2 + \log_{8} 9 = \frac{1}{3}\log_{2} 2 + \frac{2}{3}\log_{2} 3 \Rightarrow m = \frac{1}{3} + \frac{2}{3}\log_{2} 3 \Rightarrow (m - \frac{1}{3}) \times \frac{3}{2} = \log_{2} 3$ ☆

$\log_{4} 12 = \log_{4} 3 + \log_{4} 4 = 1 + \log_{4} 3 = 1 + \frac{1}{2}\log_{2} 3 \overset{☆}{\Rightarrow} \log_{4} 12 = 1 + \frac{1}{2} \times \frac{3}{2}(m - \frac{1}{3}) \Rightarrow$ 2

$\log_{4} 12 = 1 + \frac{3}{4}m - \frac{1}{4} \Rightarrow \log_{4} 12 = \frac{3}{4} + \frac{3}{4}m = \frac{3}{4}(1+m)$

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(8)

$(\frac{2}{5})^{2x-1} = (\frac{5}{2})^{3x^2} \Rightarrow (\frac{5}{2})^{1-2x} = (\frac{5}{2})^{3x^2} \Rightarrow 1 - 2x = 3x^2 \Rightarrow 3x^2 + 2x - 1 = 0 \Rightarrow x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0 \rightarrow x = \frac{-3}{3} = -1$ غ ق ق

$\qquad\qquad\quad\; \rightarrow x = \frac{1}{3}$ ق ق

$9x + 1 > 0 \Rightarrow x > \frac{1}{9}$

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$\log_{8}(4) = \frac{2}{3}\log_{2} 2 = \frac{2}{3}$

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(9)

$\log_{2} 3 = a$, $\log_{8} b = \frac{2}{3}(1+a) \Rightarrow \log_{8} b = \frac{2}{3}(\log_{2} 2 + \log_{2} 3) \Rightarrow \frac{2}{3}(\log_{2} 6) \Rightarrow \log_{8} 36 = \log_{8} b$ 2

$b = 36 \Rightarrow \log(3b - 8) \xrightarrow{b=36} \log 100 = 2$

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(10)

$-4ax^2 + bx + \frac{1}{2}c = 0 \longrightarrow$ جمع ریشه‌ها: $\frac{-b}{-4a} \Rightarrow \frac{4a}{b} = \log 4 \Rightarrow \frac{b}{a} = \log_{4} 10^4 = \log_{2} 10^2$

$\frac{b+c}{2} = a$, $2a = b + c \Rightarrow 2 = \frac{b}{a} + \frac{c}{a} \Rightarrow \frac{c}{a} = 2 - \log_{2} 100 = 2 - 2(\log_{2} 5 + 1)$ 2 $\frac{c}{a} = -2\log_{2} 5$

$(\frac{1}{\sqrt[8]{2}})^{-2\log_{2} 5} \Rightarrow (2^{-\frac{1}{8}})^{-2\log_{2} 5} \Rightarrow 2^{\frac{1}{4}\log_{2} 5} \Rightarrow 2^{\log_{2}\sqrt[4]{5}} \Rightarrow \sqrt[4]{5}$