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نام و نام خانوادگی ...نازنین کریمی... پاسخنامه تشریحی تکلیف شماره ...۲۳... کلاس ...یازدهم A...

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$$\log_{mn} m^2n = b \longrightarrow \frac{\log_n m^2n}{\log_n mn} = b \longrightarrow \frac{\log_n m^2 + \log_n n}{\log_n m + \log_n n} = b \longrightarrow$$

$$\frac{2\log_n m + 1}{\log_n m + 1} \xrightarrow{\log_n m = a} \frac{2a+1}{a+1} = b \Rightarrow b = \underbrace{2 - \frac{1}{a+1}}_{\text{بین ۱ و ۲}} \xrightarrow{0<a<1} [b] = [2^-] = 1$$

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الف) $\sqrt{\frac{x}{\log_{\frac12} x}} \Rightarrow \frac{x}{\log_{\frac12} x} \geq 0 \xrightarrow{x>0 \ (1)} \log_{\frac12} x > 0 \Rightarrow x<1 \ (2) \to 1\cap 2 \Rightarrow x \in (0,1)$

ب) $\frac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1} \Rightarrow x^2-x-2>0 \Rightarrow (x+1)(x-2)>0 \to \ + \ -1 \ - \ 2 \ + \quad (1)$

$x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow x \geq 1 \ , \ x \leq -1 \quad (2)$

$1\cap 2 \Rightarrow \quad \Rightarrow (-\infty, -1) \cup (2, +\infty)$

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$$2\log_9 a + \log_a \sqrt{9} = 2 \Rightarrow 2\log_{3^2} a + \log_a 3 = 2 \Rightarrow \log_3 a + \log_a 3 = 2 \Rightarrow$$

$$\log_3 a + \frac{1}{\log_3 a} = 2 \xrightarrow{\log_3 a = t} t + \frac{1}{t} = 2 \Rightarrow t^2 - 2t + 1 = 0 \Rightarrow (t-1)^2 = 0 \Rightarrow t = 1$$

$$\log_3 a = 1 \Rightarrow \boxed{a} = 3^1 = \boxed{3}$$

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$$\left(\log \frac{5}{3}\right)x^2 + (\log 9)x - \log 15 = 0 \Rightarrow (\overset{0.7}{\log 5} - \overset{0.4}{\log 3})x^2 + (\overset{0.8}{2\log 3})x - (\overset{0.7}{\log 5} + \overset{0.4}{\log 3}) = 0$$

$$\log 2 = 0.3 \Rightarrow \log \frac{10}{5} = 0.3 \Rightarrow \overset{1}{\log 10} - \log 5 = \frac{3}{10} \Rightarrow \boxed{\log 5 = 0.7} \Rightarrow 0.3x^2 + 0.8x - 1.1 = 0$$

اختلاف ریشه‌ها

$$3x^2 + 8x - 11 = 0 \Rightarrow (x-1)(3x+11) = 0 \to x \begin{cases} 1 \\ -\frac{11}{3} \end{cases} \Rightarrow 1 - \left(-\frac{11}{3}\right) \Rightarrow \frac{14}{3}$$

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$\log_2 7 = 2.8$

$\log_5 2 = 0.5$

$\log_{14} 10 = ?$

$\boxed{\log_2 5 = \frac{1}{\log_5 2} = \frac{1}{0.5} = 2}$

$$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 5 + \log_2 2}{\log_2 7 + \log_2 2} = \frac{\log_2 5 + 1}{\log_2 7 + 1} = \frac{2+1}{2.8+1} \Rightarrow \frac{3}{3.8} = \frac{30}{38} = \boxed{\frac{15}{19}}$$

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$$\frac{\log_3 6}{\log_3 15} = \frac{\log_3 2 + \log_3 3}{\log_3 5 + \log_3 3} = \frac{\frac{1}{1.6} + 1}{1.5 + 1} = \frac{13}{20} \quad (۶$$

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$$\left.\begin{array}{l}\log_3 5 = 1.5\\ \log_2 3 = 1.6\end{array}\right\} \xrightarrow{\text{ضرب}} \log_3 5 \times \log_2 3 \Rightarrow \frac{\log 5}{\cancel{\log 3}} \times \frac{\cancel{\log 3}}{\log 2} = \frac{\log 5}{\log 2} = \log_2 5 = 1.5 \times 1.6 = \boxed{2.4}$$

$$\boxed{\log_5 2} = \frac{1}{2.4} = \frac{1}{\frac{24}{10}} \Rightarrow \frac{10}{24} = \boxed{\frac{5}{12}}$$

$$\log_{15} 6 = ? \Rightarrow \frac{\log 2}{\cancel{\log 3}} \times \frac{\cancel{\log 3}}{\log 5} = \frac{\log 2}{\log 5} = \boxed{\log_5 2 = \frac{5}{12}} \simeq \boxed{0.4}$$

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$$\log_8 18 = m,\quad \log_4 12 = ?$$

$$\log_8 18 = \frac{\log 18}{\log 8} = \frac{2\log 3 + \log 2}{3\log 2} = m \Rightarrow \frac{\log 3}{\log 2} = \frac{3m-1}{2}$$

$$\log_4 12 = 1 + \log_4 3 = 1 + \frac{1}{2}\left(\frac{3m-1}{2}\right) = \boxed{\frac{3}{4}(m+1)}$$

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$$(0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \xrightarrow[\frac{4}{10}=\frac{2}{5}]{\frac{125}{8}=\left(\frac{5}{2}\right)^3} \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \xRightarrow{\left(\frac{5}{2}\right)=\left(\frac{2}{5}\right)^{-1}}$$

$$\left(\frac{2}{5}\right)^{2x-1} = \left(\frac{2}{5}\right)^{-3x^2} \Longrightarrow 2x-1 = -3x^2 = 3x^2+2x-1 = 0 \Rightarrow (3x-1)(x+1) = 0$$

$$x = \frac{1}{3} \checkmark \qquad x = -1 \text{ غ ق ق}$$

$$\log_8 (9x+1) \Rightarrow 9x+1 > 0 \Rightarrow x > -\frac{1}{9}$$

برای $x=-1$، $9x+1$ درست نیست.

$$\log_8 (3+1) \Rightarrow \frac{2}{3}\log_2 2 = \frac{2}{3} \Leftarrow \text{جواب}$$

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$$\log_2 3 = a \Rightarrow 2^a = 3,\quad \log_8 b = \frac{2}{3}(1+a) \Rightarrow 2^{2(1+a)} = b$$

$$\left(2^{1+a}\right)^2 = b \Rightarrow (2\times 2^a)^2 = b \Rightarrow (2\times 3)^2 = b \Rightarrow \boxed{b = 36}$$

$$\log(3b-8) = \log(3\times 36 - 8) = \log 100 = \boxed{2}$$

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$$2a = b + c \Rightarrow b = 2a - c \Rightarrow \frac{1}{x_1+x_2} = \frac{1}{s} = \frac{4a}{b} = \frac{4a}{2a-c} = \log 4 = 2\log 2$$

$$\frac{2a}{2a-c} = \log 2 \Rightarrow \frac{2a-c}{2a} = \log_2 10 \Rightarrow 1 - \log_2 10 = \frac{c}{2a} \Rightarrow \boxed{2 - \log_2 100 = \frac{c}{a}}$$

$$\left(\sqrt[8]{\frac{1}{2}}\right)^{\frac{c}{a}} = \left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}} = \left(2^{2-\log_2 100}\right)^{-\frac{1}{8}} = \left(\frac{2^2}{2^{\log_2 100}}\right)^{-\frac{1}{8}} = \left(\frac{4}{100}\right)^{-\frac{1}{8}} = (25)^{\frac{1}{8}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$$

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