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$$b = \log_{mn} m^n = \frac{\log_n m^n}{\log_n mn} = \frac{\log_n m^n + \log_n n}{\log_n m^n + \log_n n} = \frac{2 \log_n m + 1}{\log_n m + 1} = \frac{2a+1}{a+1} = 1 + \frac{a}{a+1}$$

$\downarrow$   
 $\frac{a}{a+1} < 1$

$$\Rightarrow \left[ 1 + \frac{a}{a+1} \right] = \boxed{1}$$

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الف)  $\sqrt{\frac{x}{\log \frac{1}{x}}} \Rightarrow \frac{x}{\log \frac{1}{x}} \geq 0 \Rightarrow \log \frac{1}{x} > 0 \Rightarrow x \in (0, 1)$

ب)  $x^2 - x - 2 > 0 \Rightarrow \frac{-1 \pm \sqrt{1+8}}{2} \Rightarrow (-\infty, -1) \cup (2, +\infty)$

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$$2 \log_{\frac{1}{x}} a + \log_a \sqrt{a} = 2 \Rightarrow \log_{\frac{1}{x}} a + \log_{\frac{1}{x}} a = 2 \Rightarrow \log_{\frac{1}{x}} a = 1 \Rightarrow \frac{1}{\log_x a} = 1 \Rightarrow \log_x a = 1$$

$$\Rightarrow x = \frac{1}{a} = 2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 - 4}}{2} = 1 \Rightarrow \log_x a = 1 \Rightarrow \boxed{x = 2}$$

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$$(\log^a - \log^b) m^n + (2 \log^b) m - (\log^a - \log^b) = 0$$

$$\Rightarrow (\log^a - \log^b - \log^b) m^n + (2 \log^b) m - (\log^a - \log^b) = 0$$

$$1 - \log^b - \log^b \quad \quad \quad \log^b \quad \quad \quad \log^b \in 1 - \log^b$$

$$\Rightarrow \log^b m^n + \log^b m - \log^b = 0 \Rightarrow \frac{\sqrt{a}}{|a|} = \frac{1}{\log^b} = \boxed{\frac{1}{\log^b}}$$

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$$\frac{1}{\log^a} = \frac{1}{r} \Rightarrow \log^a = r$$

$$\frac{\log^b}{\log^b} = \frac{\log^b + \log^b}{\log^b + \log^b} = \frac{2}{2} = \boxed{\frac{15}{19}}$$



$$\log_{10} \frac{1}{10} = \frac{\log_{10} \frac{1}{10}}{\log_{10} \frac{1}{10}} = \frac{\log_{10} 1 - \log_{10} 10}{\log_{10} 1 - \log_{10} 10} = \frac{0 - 1}{-1 + 1} = \frac{-1}{0} \quad \boxed{\text{undefined}}$$

$$\log_r r = \frac{1}{\log_r r} = 1 \Rightarrow \frac{1}{1} = \log_r r = \frac{1}{1}$$

$$\log_n n = m \Rightarrow \log_r n = \frac{1}{r} \log_n n = m \Rightarrow \log_r n = rm \Rightarrow \log_r r + \log_r n = rm \quad - \quad$$

$$\Rightarrow 1 + r \log_r n = rm \Rightarrow \frac{rm-1}{r} = \log_r n$$

$$\log_r r = \log_r r + \log_r 1 = \frac{1}{r} \log_r r + 1 \Rightarrow \frac{1}{r} \left( \frac{rm-1}{r} \right) + 1 = \boxed{\frac{r}{r} (m+1)}$$

$$\left( \frac{r}{\omega} \right)^{m-1} = \left( \frac{10}{\omega} \right)^{m-1} \Rightarrow \frac{10}{\omega} \left( \frac{10}{\omega} \right)^{m-1} = \left( \frac{r}{\omega} \right)^{m-1} \Rightarrow \left( \frac{10}{\omega} \right)^m = \left( \frac{r}{\omega} \right)^{m-1}$$

$$\Rightarrow \frac{10^m}{\omega^m} = \frac{r^{m-1}}{\omega^{m-1}} \Rightarrow 10^m = r^{m-1} \omega \Rightarrow m \log 10 = (m-1) \log r + \log \omega$$

$$\frac{1}{r} = n \Rightarrow \log_n (n+1) = \log_r r = \frac{1}{r} \log_r r = \boxed{\frac{1}{r}}$$

$$\log (r^b - 1) = ? \quad \log \frac{b}{a} = \frac{r}{a} (1+a) \log_r r = a$$

$$\boxed{\log (1.1 - 1) = 0} \quad \frac{1}{r} \log_r b = \frac{r}{\omega} (1+a)$$

$$r(1 + \log_r r) = r \log_r r = \log_r r = \log_r r = b \log_r r$$

$$\frac{f_a}{b} = r \log^r \Rightarrow \frac{r_a}{b} = \log^r \rightarrow b = \frac{r_a}{\log^r}$$

$$\frac{b+d}{r} = a \rightarrow c = ra - b$$

$$\frac{c}{r} = \frac{ra-b}{a} = r - \frac{b}{a} = r - \left( \frac{ra}{\log^r} \times \frac{1}{a} \right) = r - \frac{r}{\log^r} = r \left( 1 - \frac{1}{\log^r} \right)$$

$$r - r \log^r = r - \log^r$$

$$\frac{1}{\log^r} = \left( r^{-\frac{1}{r}} \right)^{r-1} \log^r = (r^{-\frac{1}{r}})^{r-1} \log^r$$

$$\Rightarrow r^{-\frac{1}{r}} \log^r = r \log^r \Rightarrow \omega \log_r r = \boxed{\frac{1}{r} \log_r r}$$