ثبت ۲۳ — یازدهم دختران ۸ — ۲۰ — سانیا ماریان

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$$b=\log_{mn}^{m^2n}=\frac{\log_n^{m^2n}}{\log_n^{mn}}=\frac{\log_n^{m^2}+\log_n^n}{\log_n^m+\log_n^n}=\frac{2\log_n^m+1}{\log_n^m+1}=\frac{2a+1}{a+1}=1+\frac{a}{a+1}$$

$$0<\frac{a}{a+1}<1$$

$$\Rightarrow \left[1+\frac{a}{a+1}\right]=\boxed{1}$$

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الف) $\sqrt{\dfrac{x}{\log_{\frac12}^x}} \Rightarrow \dfrac{x}{\log_{\frac12}^x}\geq 0 \Rightarrow \log_{\frac12}^x>0 \Rightarrow x\in(0,1)$

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ب) $x^2-x-2>0 \Rightarrow \begin{array}{ccccc} & -1 & & 2 & \\ + & | & - & | & + \end{array}$

$x^2-1>0 \Rightarrow \begin{array}{ccccc} & -1 & & 1 & \\ + & | & - & | & + \end{array}$

$\Big\}\ (-\infty,-1)\cup(2,+\infty)$

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$$2\log_{3^2}^{a}+\log_a^{\sqrt9}=2 \Rightarrow \log_3^a+\log_a^3=2 \Rightarrow \log_3^a+\frac{1}{\log_3^a}=2$$

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$$\Rightarrow t+\frac1t=2 \Rightarrow t^2-2t+1=0 \Rightarrow \frac{-b}{2a}=\frac22=1 \Rightarrow 1=\log_3^a \Rightarrow \boxed{a=3}$$

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$$(\log^5-\log^3)x^2+(2\log^3)x-(\log^{30}-\log^2)=0$$

$$\Rightarrow (\log^{10}-\log^2-\log^3)x^2+(2\log^3)x-(\log^3+\log^{10}-\log^2)=0$$

$1-0.3-0.4 \qquad 0.8 \qquad 0.4+1-0.3$

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$$\Rightarrow 0.3x^2+0.8x-1.1=0 \Rightarrow \frac{\sqrt\Delta}{|a|}=\frac{1.4}{0.3}=\boxed{\frac{14}{3}}$$

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$$\frac{1}{\log_5^2}=\frac12 \Rightarrow \log_2^5=2 \qquad \frac{\log_2^{10}}{\log_2^{12}}=\frac{\overset{2}{\log_2^{5}}+\overset{1}{\log_2^{2}}}{\underset{2.8}{\log_2^{6}}+\underset{1}{\log_2^{2}}}=\frac{3}{3.8}=\boxed{\frac{15}{19}}$$

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$$\log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{\log_3 3+\log_3 2}{\log_3 5+\log_3 3} = \frac{\frac{5}{8}+1}{\frac{3}{2}+1} = \frac{13}{20}\ \boxed{0.65}$$

$$\log_2 3 = \frac{1}{\log_3 2} = 1.6 \Rightarrow \frac{10}{16} = \log_3 2 = \frac{5}{8}$$

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$$\log_8 18 = m \Rightarrow \log_{2^3} 18 = \frac{1}{3}\log_2 18 = m \Rightarrow \log_2 18 = 3m \Rightarrow \log_2 2+\log_2 3^2 = 3m$$

$$\Rightarrow 1+2\log_2 3 = 3m \Rightarrow \frac{3m-1}{2} = \log_2 3$$

$$\log_4 12 = \log_4 3+\log_4 4 = \frac{1}{2}\log_2 3+1 \Rightarrow \frac{1}{2}\left(\frac{3m-1}{2}\right)+1 = \boxed{\frac{3}{4}(m+1)}$$

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$$(0.4)^{2n-1} = \left(\frac{125}{8}\right)^{n^2} \Rightarrow \frac{125}{8} = \left(\frac{5}{2}\right)^3 \qquad \left(\frac{2}{5}\right)^{2n-1} = \left(\left(\frac{5}{2}\right)^3\right)^{n^2}$$

$$\frac{4}{10} = \frac{2}{5}$$

$$\Rightarrow \left(\frac{2}{5}\right)^{2n-1} = \left(\frac{2}{5}\right)^{-3n^2}$$

$$\Rightarrow 2n-1 = -3n^2 \Rightarrow 3n^2+2n-1 = 0$$

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$$\frac{1}{3} = n \Rightarrow \log_8 (9n+1) = \log_8 4 = \frac{2}{3}\log_2 2 = \boxed{\frac{2}{3}}$$

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۹)

$$\log(3b-8) = ? \qquad \log_8 b = \frac{2}{3}(1+a) \quad \log_2 3 = a$$

$$\boxed{\log(108-8) = 2} \qquad \frac{1}{3}\log_2 b = \frac{2}{3}(1+a)$$

$$2(1+\log_2 3) = 2\log_2 6 = \log_2 36 = \log_2 b \Rightarrow b = 36$$

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$$\frac{4a}{b} = 2\log 2 \Rightarrow \frac{2a}{b} = \log 2 \to b = \frac{2a}{\log 2}$$

$$\frac{b+c}{2} = a \to c = 2a-b$$

$$\frac{c}{a} = \frac{2a-b}{a} = 2-\frac{b}{a} = 2-\left(\frac{2a}{\log 2}\times\frac{1}{a}\right) = 2-\frac{2}{\log 2} = 2\Big(1-\cancel{\frac{1}{\log 2}}\Big)$$

$$\log_2 10$$

$$2-2\log_2 10 = 2-\log_2 100 \qquad \frac{1}{\sqrt[8]{2}} = \left(2^{-\frac{1}{8}}\right)^{2-2\log_2 10} = \left(2^{-\frac{1}{8}}\right)^{2(1-\log_2 10)}$$

$$\log_2 0.2$$

$$\Rightarrow 2^{-\frac{1}{4}\log_2 0.2} = 2^{\log_{2^4} 5} \Rightarrow 5^{\log_{2^4} 2} = 5^{\frac{1}{4}\log_2 2} = \boxed{\sqrt[4]{5}}$$