تکلیف شماره ۳ ۲۰ نجاء موسوی یازدهم دفتر A

① $\log_n m = a \quad (a>0) \qquad \log_{mn} m^2 n = b$

$[b] = ?$

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$$\log_{mn} m^2 n = \log_{mn} mn + \log_{mn} m = 1 + \log_{mn} m$$

$$= 1 + \frac{1}{\log_m mn} = 1 + \frac{1}{\log_m m + \log_m n} = 1 + \underbrace{\frac{1}{1+\frac{1}{a}}}_{\text{جواب کسر کوچکتر از یک میشه}} = b$$

$$[b] = 1 + \underbrace{\left[\frac{1}{1+\frac{1}{a}}\right]}_{0} = 1$$

② الف) $y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}} \Rightarrow$

$\log_{\frac{1}{2}} x = 0 \to x = 1$

$x > 0 \,,\ x \neq 1$

$\dfrac{x}{\log_{\frac{1}{2}} x} \geq 0 \qquad \begin{array}{ccccc} & 0 & & 1 & \\ - & & + & & - \end{array} \Rightarrow D_f = (0,1)$ ۵

ب) $y = \dfrac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1} \qquad ① \cap ② \Rightarrow D_f = (-\infty,-1) \cup (2,+\infty)$

$x^2-x-2>0 \to (x-2)(x+1)>0 \qquad \begin{array}{ccccc} & -1 & & 2 & \\ + & & - & & + \end{array} \qquad (-\infty,-1)\cup(2,+\infty)$ ①

$\sqrt{x^2-1}+1 \geq 0$

$x^2-1 \geq 0 \to x \geq 1 \quad x \leq -1 \qquad \begin{array}{ccccc} & -1 & & 1 & \\ + & & - & & + \end{array} \qquad (-\infty,-1] \cup [1,+\infty)$ ②

③

$$2\log_x^{a} + \log_a^{\sqrt{x}} = 2 \qquad x=9$$

$$2\log_9^{a} + \log_a^{3} = 2$$

$$2\log_{3^2}^{a} + \frac{1}{\log_3^{a}} = 2$$

$$\log_3^{a} + \frac{1}{\log_3^{a}} = 2 \rightarrow t + \frac{1}{t} = 2$$

$$t^2 + 1 - 2t = 0$$

$$t^2 - 2t + 1 = 0$$

$$(t-1)^2 = 0 \rightarrow t = 1$$

$$\log_3^{a} = 1 \rightarrow \boxed{a = 3}$$

④ $\log 3 \simeq 0{,}4 \,,\quad \log 2 \simeq 0{,}3$

$$\left(\log^{\frac{5}{3}}\right)x^2 + (\log 9)\,x - \log 15 = 0$$

$$\left(\log^{5} - \log^{3}\right)x^2 + \left(2\log^{3}\right)x - \left(\log^{3} + \log^{5}\right) = 0$$

$$a+b+c=0 \rightarrow x=1$$

$$\hookrightarrow x = \frac{c}{a} = \frac{-(\log^{3} + \log^{5})}{\log^{5} - \log^{3}} =$$

اختلاف ریشه‌ها $= 1 + \frac{11}{3} = \boxed{\frac{14}{3}}$

$$\frac{-\left(\log^{3} + \log^{\frac{10}{2}}\right)}{\log^{\frac{10}{2}} - \log^{3}} = \frac{-\log^{3} - \log^{10} + \log^{2}}{\log^{10} - \log^{2} - \log^{3}} = \frac{-0{,}4 - 1 + 0{,}3}{1 - 0{,}3 - 0{,}4}$$

$$= \frac{-1{,}1}{0{,}3} = \frac{-11}{3}$$

تاریخ: / /

$$\log_2 ۵ = \frac{10}{5} = 2$$

(۵) $\log_2 3 = 1.5$ ، $\log_2 7 = 2.8$

$$\log_{14} 10 = ?$$

$$\log_{14 = 2\times 7} 10 = 2\times 5 = \frac{\log_2 10}{\log_2 14} = \frac{\log_2 2 + \log_2 5}{\log_2 2 + \log_2 7}$$

$$= \frac{1+2}{1+2.8} = \frac{3}{3.8} = \frac{3\times 10}{38} = \frac{30}{38} = \boxed{\frac{15}{19}}$$

$$\log_3 2 = \frac{10}{16}$$

(۶) $\log_2 3 = 1.6$ ، $\log_3 5 = 1.5$

$$\log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{\log_3 3 + \log_3 2}{\log_3 3 + \log_3 5}$$

$$= \frac{1+\frac{10}{16}}{1+1.5} = \frac{\frac{13}{8}}{2.5} = \frac{13\times 2}{8\times 5} = \boxed{\frac{13}{20}}$$

(۷) $\log_8 18 = m$

$$\log_4 12 =$$

$$\log_8 18 = \log_{2^3} 18 = \frac{1}{3}\log_2 18 = \frac{1}{3}(\log_2 2 + \log_2 9) =$$

$$\frac{1}{3}(1 + 2\log_2 3) = \frac{1}{3} + \frac{2}{3}\log_2 3 = m$$

$$\log_2 3 = \frac{(m-\frac{1}{3})3}{2} = \frac{3m-1}{2}$$

$$\log_{٤}^{١٢} = \log_{٢^٢}^{١٢} = \frac{١}{٢}\log_{٢}^{١٢} = \frac{١}{٢}\left(٢\log_{٢}^{٢} + \log_{٢}^{٣}\right)$$

$$= \frac{١}{٢}\left(٢ + \log_{٢}^{٣}\right) = ١ + \frac{١}{٢}\left(\frac{٣m-١}{٢}\right) = ١ + \frac{٣m-١}{٤}$$

$$= ١ + \frac{٣m}{٤} - \frac{١}{٤} = \frac{٣m}{٤} + \frac{٣}{٤} = \frac{٣}{٤}(m+١)$$

$$(٠/٤)^{٢x-١} = \left(\frac{١٢٥}{٨}\right)^{x^٢} \qquad \log_{٨}^{(٩x+١)}$$ ٨

$$\left(\frac{٢}{٥}\right)^{٢x-١} = \left(\frac{٥^٣}{٢^٣}\right)^{x^٢} = \left(\frac{٥}{٢}\right)^{٣x^٢}$$

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$$\left(\frac{٥}{٢}\right)^{١-٢x} = \left(\frac{٥}{٢}\right)^{٣x^٢} \rightarrow ٣x^٢ = ١-٢x$$

$$٣x^٢ + ٢x - ١ = ٠$$

$a+c=b \Rightarrow x=-١$ غ‌ق‌ق

$$x = \frac{-c}{a} = \frac{١}{٣}$$

$$\log_{٨}^{٩\times\frac{١}{٣}+١} = \log_{٨}^{٤} = \log_{٢^٣}^{٤} = \frac{١}{٣}\log_{٢}^{٤} = \frac{١}{٣}\times٢ = \boxed{\frac{٢}{٣}}$$

$$\log_{٨=٢^٣}^{b} = \frac{٢}{٣}(١+a) \quad , \quad \log_{٢}^{٣} = a$$ ٩

$$\log(٢b-٨) = ?$$

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$$\frac{١}{٣}\log_{٢}^{b} = \frac{٢}{٣}\left(١+\log_{٢}^{٣}\right) = \frac{٢}{٣}\left(\log_{٢}^{٢} + \log_{٢}^{٣}\right)$$

$$\frac{١}{٣}\log_{٢}^{b} = \frac{٢}{٣}\log_{٢}^{٦}$$

$$\frac{١}{٣}\log_{٢}^{b} = \frac{١}{٣}\log_{٢}^{٦^٢} \Rightarrow b = ٦^٢ = ٣٦$$

$$\log(3\times 36-8)=\log 100=\boxed{2}$$

$$-4ax^2+bx+\frac{1}{2}c=0$$ 15

عکس مجموع ریشه‌های معادله بالا $=\log 4$

$$a=\frac{b+c}{2}\longrightarrow 2a=b+c$$

$$c=2a-b$$

$$\left(\frac{1}{\sqrt[8]{2}}\right)^{\frac{c}{a}}=?$$

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مجموع ریشه‌ها $=\frac{-b}{a}=\frac{-b}{-4a}=\frac{b}{4a}$

$$\frac{4a}{b}=\log 4=2\log 2\rightarrow\frac{2a}{b}=\log 2\rightarrow b=\frac{2a}{\log 2}$$

$$\frac{c}{a}=\frac{2a-b}{a}=2-\frac{b}{a}=2-\left(\frac{2}{\log 2}\right)$$

$$=2\left(1-\frac{1}{\log 2}\right)=2\left(1-\log_2 10\right)$$

$$\frac{1}{\sqrt[8]{2}}=\frac{1}{2^{\frac{1}{8}}}=2^{-\frac{1}{8}}$$

$$\left(2^{-\frac{1}{8}}\right)^{\frac{c}{a}}=\left(2^{-\frac{1}{8}}\right)^{2\left(\overset{\log_2 2}{1}-\log_2 10\right)}=\left(2^{-\frac{1}{8}}\right)^{2\left(\log_2 0.2\right)}$$

$$\left(2^{-\frac{1}{4}\log_2 0.2}=2^{\log_{16}5}=5^{\log_{16}2}=5^{\log_{2^4}2}=5^{\frac{1}{4}\log_2 2}$$

$$=5^{\frac{1}{4}}=\boxed{\sqrt[4]{5}}$$ جواب