پرنیا دیباجی   یازدهم تجربی   سه‌شنبه A

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① $\log_n m = a \Rightarrow n^a = m \qquad \log_{mn} m^2 n = b \Rightarrow \log_{n^{a+1}} n^{2a+1} = b \Rightarrow \frac{2a+1}{a+1}\log_n n = b \Rightarrow$

$b = \frac{2a+1}{a+1} \Rightarrow [b] = \left[\frac{2a+1}{a+1}\right] \Rightarrow [b] = \left[\frac{a+a+1}{a+1}\right] \Rightarrow [b] = \left[\frac{a}{a+1}+1\right]$

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$\Rightarrow [b] = \left[\frac{a}{a+1}\right]+1 \qquad a>0 ,\ 0<\frac{a}{a+1}<1 \qquad \boxed{[b] = 0+1 = 1}$

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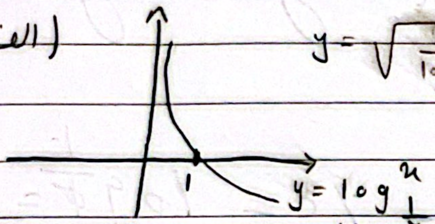
② الف)



$y = \sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$

حدود log باید ⊕ باشد پس $x>0$ است

زیر رادیکال باید بزرگتر و یا مساوی صفر باشد و با توجه به

⊕ بودن x باید $\log_{\frac{1}{2}} x$ مثبت و در نتیجه $0<x<1$

باشد.

از اشتراک مقادیر به دست آمده برای x حدود آن به صورت $0<x<1$ است که شامل هیچ عدد صحیح نمی‌باشد

ب) $y = \frac{\log_2 (x^2-x-2)}{\sqrt{x^2-1}+1}$ $\qquad \sqrt{x^2-1}+1 \neq 0 ,\ x^2-1 \geq 0 ,\ x^2-x-2>0$

$\Rightarrow x^2-x-2>0$

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$x \Rightarrow x<-1$ یا $x>2$

$x^2-1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow \begin{cases} x \geq 1 \\ \text{یا} \\ x \leq -1 \end{cases} \qquad \sqrt{x^2-1}+1 \neq 0 \Leftarrow$ همواره برقرار است

اشتراک $\Leftarrow \boxed{(-\infty, -1) \cup (2, +\infty)}$

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③ $2\log_x a + \log_a \sqrt{x} = 2 \Rightarrow 2\log_{\underbrace{9}_{3^2}} a + \log_a \sqrt{9} = 2 \Rightarrow \log_3 a + \log_a 3 = 2$

$x=9$

$a=?$

$\log_3 a + \frac{1}{\log_3 a} = 2 \Rightarrow t + \frac{1}{t} = 2$

$\log_3 a = t$

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$\xrightarrow{\times t} t^2 - 2t + 1 = 0 \Rightarrow \{x=1$

$\Rightarrow \log_3 a = 1 \Rightarrow a = 3^1 = \boxed{3}$

(4)

$\log 2 \simeq 0.3$

$\log 3 \simeq 0.4$

$$\left(\log \frac{5}{3}\right)x^2 + (\log 9)x - \log 15 = 0$$

$$\Rightarrow (\log 5 - \log 3)x^2 + (2\log 3)x - \log 5 - \log 3 = 0$$

$$\Rightarrow (\log 5)x^2 - 0.4x^2 + 0.8x - \log 5 - 0.4 = 0$$

$$\Rightarrow \left(\log \frac{10}{2}\right)x^2 - 0.4x^2 + 0.8x - \left(\log \frac{10}{2}\right) - 0.4 = 0$$

$$\Rightarrow (\log 10 - \log 2)x^2 - 0.4x^2 + 0.8x - (\log 10 - \log 2) - 0.4 = 0$$

$$\Rightarrow (1 - 0.3)x^2 - 0.4x^2 + 0.8x - (1 - 0.3) - 0.4 = 0$$

$$\Rightarrow 0.3x^2 + 0.8x - 1.1 = 0 \Rightarrow \begin{cases} x = \frac{-11}{3} \\ x = 1 \end{cases}$$

$$\Rightarrow 1 - \left(-\frac{11}{3}\right) = \boxed{\frac{14}{3}}$$

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(5)

$\log_2 7 = 2.8 \qquad \log_5 2 = 0.5 \qquad \log_{14} 10 = ?$

$$\frac{\log 2}{\log 5} = \frac{1}{2} \Rightarrow \frac{\log 2}{\log \frac{10}{2}} = \frac{1}{2} \Rightarrow \frac{\log 2}{\log 10 - \log 2} = \frac{1}{2} \Rightarrow \frac{\log 2}{1 - \log 2} = \frac{1}{2} \Rightarrow \log 2 = \frac{1}{3}$$

$$\frac{\log 7}{\log 2} = \frac{28}{10} \Rightarrow \frac{\log 7}{\frac{1}{3}} = \frac{28}{10} \Rightarrow \log 7 = \frac{14}{15}$$

$$\frac{1}{\log_{10} 14} = \frac{1}{\log 2 \times 7} = \frac{1}{\log 2 + \log 7}$$

$$\Rightarrow \frac{1}{\frac{1}{3} + \frac{14}{15}} = \boxed{\frac{15}{19}}$$

(۶)

$\log_{3}^{5} = 1.5 \qquad \log_{2}^{3} = 1.6 \qquad \log_{15}^{6} = ?$

$\log_{3}^{5} \times \log_{2}^{3} = \log_{2}^{5} \qquad \log_{3}^{5} \times \log_{2}^{3} = 1.5 \times 1.6 \Rightarrow \log_{2}^{5} = 2.4$

$\log_{15}^{6} = \frac{\log_{2}^{6}}{\log_{2}^{15}} \Rightarrow \log_{15}^{6} = \frac{\log_{2}^{2} + \log_{2}^{3}}{\log_{2}^{3} + \log_{2}^{5}} = \frac{1+1.6}{1.6+2.4} = \frac{2.6}{4} = \boxed{0.65}$

(۷)

$\log_{8}^{18} = m \qquad \log_{4}^{12} = \frac{\log_{8}^{12}}{\log_{8}^{4}}$

$\log_{4}^{12} = ?$

$\log_{8}^{12} = \log_{8}^{4} + \log_{8}^{3} = \log_{2^3}^{2^2} + \log_{2^3}^{3} =$

$\Rightarrow \frac{2}{3}\log_{2}^{2} + \frac{1}{3}\log_{2}^{3} = \frac{2}{3} + \frac{1}{3}\log_{2}^{3}$

$\log_{8}^{4} = \log_{2^3}^{2^2} = \frac{2}{3}\log_{2}^{2} = \frac{2}{3}$

$\log_{8}^{18} = m \Rightarrow \log_{8}^{9} + \log_{8}^{2} = m \Rightarrow \log_{2^3}^{3^2} + \log_{2^3}^{2} = m \Rightarrow \frac{2}{3}\log_{2}^{3} + \frac{1}{3}\log_{2}^{2} = m$

$\Rightarrow \frac{2}{3}\log_{2}^{3} + \frac{1}{3} = m \Rightarrow \log_{2}^{3} = \frac{3m-1}{2}$

$\log_{4}^{12} = \frac{\frac{2}{3} + \frac{1}{3}\log_{2}^{3}}{\frac{2}{3}} = \frac{\frac{2}{3} + \frac{1}{3}\left(\frac{3m-1}{2}\right)}{\frac{2}{3}} = \frac{2 + \left(\frac{3m-1}{2}\right)}{2} = \frac{4+3m-1}{2}$

$= \frac{3m+3}{4} = \boxed{\frac{3}{4}(m+1)}$

(۸)

$0.4 = \frac{2}{5} \qquad (0.4)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \Rightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\left(\frac{5}{2}\right)^3\right)^{x^2}$

$\Rightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow \left(\frac{5}{2}\right)^{-2x+1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow 3x^2 = -2x+1$

$\Rightarrow 3x^2+2x-1 = 0 \Rightarrow \begin{cases} x = -1 \rightarrow \text{غ ق ق} \\ x = \frac{1}{3} \end{cases}$

$\log_{8}\left(9\left(\frac{1}{3}\right)+1\right) = \log_{8}^{4} = \log_{2^3}^{2^2}$

$\Rightarrow \boxed{\frac{2}{3}\log_{2}^{2} = \frac{2}{3}}$

$\log_2 3 = a \qquad \log_8 b = \frac{2}{3}(1+a) \qquad \log(3b-8)=?$ (9)

$\log_8 b = \frac{2}{3}\left(1+\log_2 3\right) \Rightarrow$

$\log_8 b = \frac{2}{3} + \frac{2}{3}\log_2 3 \Rightarrow \log_8 b = \frac{2}{3} + \log_8 9$

$\Rightarrow \log_8 b - \log_8 9 = \frac{2}{3} \Rightarrow \log_8 \frac{b}{9} = \frac{2}{3}$

$\Rightarrow \frac{b}{9} = 8^{\frac{2}{3}} \Rightarrow \frac{b}{9} = \sqrt[3]{8^2} \Rightarrow \frac{b}{9} = 4 \Rightarrow b = 36$

$\Rightarrow \log(108-8) = \log 100 = \boxed{2}$

$-4ax^2 + bx + \frac{1}{2}c = 0 \qquad \frac{1}{\frac{-b}{-4a}} = \log 4 \Rightarrow \frac{4a}{b} = \log 4$ (10)

$\Rightarrow \frac{a}{b} = \frac{\log 4}{4} = \frac{2\log 2}{4} = \frac{\log 2}{2} \qquad 2a = b + c \Rightarrow 2\frac{a}{b} = 1 + \frac{c}{b}$

$\xrightarrow{\frac{a}{b} = \frac{\log 2}{2}} 2\left(\frac{\log 2}{2}\right) = 1 + \frac{c}{b} \Rightarrow \frac{c}{b} = -1 + \log 2$

$\begin{cases} \frac{c}{b} = -1 + \log 2 \\ \frac{a}{b} = \frac{\log 2}{2} \end{cases} \Rightarrow \frac{c}{a} = \frac{-1+\log 2}{\frac{\log 2}{2}} \Rightarrow \frac{c}{a} = \frac{\log 2 - \log 10}{\frac{1}{2}\log 2} = \frac{\log \frac{2}{10}}{\frac{1}{2}\log 2}$

$= \frac{\log \frac{1}{5}}{\frac{1}{2}\log 2} = \frac{\log \frac{1}{5}}{\log\sqrt{2}} = \log_{\sqrt{2}} \frac{1}{5} \qquad \left(\frac{1}{\sqrt[4]{2}}\right)^{\frac{c}{a}} = \left(\frac{1}{\sqrt{2}}\right)^{\log_{\sqrt{2}} \frac{1}{5}} = \left(\frac{1}{5}\right)^{\log_{\sqrt{2}} \frac{1}{\sqrt{2}}}$

$= \left(\frac{1}{5}\right)^{\log_{2^{\frac{1}{2}}} 2^{-\frac{1}{8}}} = \left(\frac{1}{5}\right)^{\left(\frac{-\frac{1}{8}}{\frac{1}{2}}\right)\log_2 2} = \left(\frac{1}{5}\right)^{-\frac{1}{4}} = 5^{\frac{1}{4}} = \sqrt[4]{5}$