۲۰ سینا شید محمدی

-۱

$\log_n m = a \quad \log_{mn} m^2 n = b \quad a>0 \quad [b]=?$

$m = n^a$

$\Rightarrow \log_{n^{a+1}} n^{(2a+1)} = b \Rightarrow \frac{2a+1}{a+1}\log_n n = \frac{2a+1}{a+1} = b$ (۳)

$a>0 \Rightarrow 1<\frac{2a+1}{a+1}<2 \Rightarrow \boxed{[b]=1}$

-۲

الف) $y=\sqrt{\frac{x}{\log_{\frac{1}{2}} x}}$ $\quad$ I) $x>0$ $\quad$ (II) $\log_{\frac{1}{2}} x = 0 \Rightarrow x \neq 1$ $\quad$ (III) $\frac{x}{\log_{\frac{1}{2}} x} \geq 0$

$I \cap II \cap III = (0,1)$ ⇐ نقاط $x=0$ و $x=1$ = ... (۲)

ب) $y=\frac{\log_4 (x^2-x-2)}{\sqrt{x^2-1}+1}$ $\quad$ I) $x^2-x-2>0 \Rightarrow (x-2)(x+1)>0$

II) $\sqrt{x^2-1}+1 \neq 0 \Rightarrow \sqrt{x^2-1} \neq -1 \Rightarrow \mathbb{R}$

$\Rightarrow I\cap II = (-\infty,-1)\cup(2,+\infty)$

-۳

$2\log_x a + \log_a \sqrt{x} = 2 \quad x=9 \Rightarrow a=?$ (۲)

$2\log_9 a + \log_a 3 = 2 \Rightarrow \log_3 a + \log_a 3 = 2 \Rightarrow \frac{1}{\log_3 a} + \log_a 3 = 2 \Rightarrow \log_a 3 = 1$

$2\log_{3^2} a \Rightarrow \frac{2}{2}\log_3 a$ $\quad \Rightarrow \boxed{a=3}$

-۴

$\log 3 \simeq 0.4 \quad \log 2 \simeq 0.3$ $\quad \sqrt{\Delta}=\sqrt{\frac{132+64}{100}}=\frac{14}{10}$ $\quad (\log 9)^2 = (2\log 3)^2 = \frac{64}{100}$

$[\log\frac{5}{3}]x^2 + (\log 9)x - \log 15 = 0$ $\quad +4(\log 5 - \log 3)(\log 5 + \log 3)$

$a = \log 5 - \log 3 = 0.7 - 0.4 = 0.3$ $\quad$ مزدوج ⇐ $+4((\log 5)^2 - (\log 3)^2)$ (۲)

$|\alpha-\beta| = \frac{\sqrt{\Delta}}{|a|}$ $\quad \log 5 = \log\frac{10}{2} = \log 10 - \log 2 = 0.7$

$=\frac{1.4}{0.3} = \frac{14}{3}$

$\Delta = b^2 - 4ac = (\log 9)^2 - 4(\log\frac{5}{3})(-\log 15)$ $\quad 4(\frac{49}{100} - \frac{16}{100}) = \frac{132}{100}$

-۵

$\log_2 7 = 2.8 \quad \log_5 2 = 0.5 \quad \log_{14} 10 = ? \quad \frac{30}{38} = \frac{15}{19}$

$\log_{14} 10 = \frac{1}{\log_{10} 14} \Rightarrow \log_{10} 14 = \log_{10} 2 + \log_{10} 7 = \frac{2.8}{3} + \frac{1}{3} = \frac{3.8}{3} = \frac{38}{30}$ (۲)

$\log_2 7 = \frac{\log_{10} 7}{\log_{10} 2} = 2.8$ $\quad \log_5 2 = \frac{\log_{10} 2}{\log_{10} 5} = \frac{\log_{10} 2}{1-\log_{10} 2} = 0.5$

$\log_{10} 7 = \frac{2.8}{3}$

$\frac{3}{2}\log_{10} 2 = \frac{1}{2} \Rightarrow \log_{10} 2 = \frac{1}{3}$ $\quad \Rightarrow \frac{1}{2} - \frac{1}{2}\log_{10} 2 = \log_{10} 2$

PAPCO

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$\log_{۳}^{۵} = ۱/۵$ $\quad \log_{۲}^{۳} = ۱/۶ \Rightarrow \log_{۳}^{۲} = \frac{۱۰}{۱۶}$ —۶

$\log_{۱۵}^{۶} = \frac{\log_{۲}^{۶}}{\log_{۲}^{۱۵}} = \frac{\log_{۲}^{۲} + \log_{۲}^{۳}}{\log_{۲}^{۳} + \log_{۲}^{۵}} = \frac{۱+۱/۶}{۱/۶+۲/۴} = \frac{۲/۶}{۴} = \frac{۱۳}{۲۰} = ۰/۶۵$

$\log_{۲}^{۵} = \frac{\log_{۳}^{۵}}{\log_{۳}^{۲}} = \frac{۱/۵}{\frac{۱۰}{۱۶}} = ۲/۴$

$\log_{۸}^{۱۸} = m \quad \log_{۴}^{۱۲} = ? \quad \log_{۴}^{۱۲} = \frac{۱}{۲}\log_{۲}^{۳} + \frac{۱}{۲}\log_{۲}^{۴} = \frac{۳m-۱+۱}{۴}$ —۷

$\log_{۸}^{۱۸} = \frac{\log_{۴}^{۱۸}}{\log_{۴}^{۸}} = \frac{\log_{۴}^{۲} + \log_{۴}^{۹}}{\log_{۴}^{۲} + \log_{۴}^{۴}} = m \Rightarrow \frac{۳m}{۲} - \frac{۱}{۲} = \log_{۴}^{۹}$ $\quad \frac{۳m+۳}{۴}$

$\Rightarrow \frac{۳m-۱}{۲} = \log_{۲}^{۳}$

$(۰/۴)^{۲x-۱} = (\frac{۱۲۵}{۸})^{x^۲} \quad \log_{۸}^{(۹x+۱)} = ?$ —۸

$(\frac{۲}{۵})^{۲x-۱} = (\frac{۵^۳}{۲^۳})^{x^۲} \Rightarrow (\frac{۲}{۵})^{۲x-۱} = (\frac{۵}{۲})^{۳x^۲} \Rightarrow (\frac{۲}{۵})^{۲x-۱} = (\frac{۲}{۵})^{-۳x^۲}$ $\quad a>0 \quad \log_{b}^{a}$

$\log_{۸}^{(۳+۱)} = \log_{۸}^{۴} = ۲\log_{۸}^{۲} \Rightarrow ۲x-۱ = -۳x^۲ \Rightarrow ۳x^۲+۲x-۱$

$\Rightarrow x^۲+۲x-۳ = (x+۳)(x-۱) \Rightarrow x = \frac{-۳}{۳} = -۱$ ✗ $\quad x = \frac{۱}{۳}$ ✓

$\Rightarrow \frac{۲}{۳}$

—۹

$\log_{۲}^{۳} = a \quad \log_{۸}^{b} = \frac{۲}{۳}(۱+a) \quad \log(۳b-۸) = ?$

$\log_{۸}^{b} = \frac{۱}{۳}\log_{۲}^{b} \Rightarrow \log_{۲}^{b} = ۲+۲a \Rightarrow \underbrace{۲^۲}_{۴} \times \underbrace{۲^{۲a}}_{(۲^a)^۲} = b \Rightarrow b = ۳۶$

$۲^a = ۳ \Rightarrow (۲^a)^۲ = ۹ \quad \log(۳b-۸) = \log(۱۰۸-۸) = \log^{۱۰۰} = ۲ \quad ۳\times۳۶ = ۱۰۸$

$-۴ax^۲ + bx + \frac{۱}{۲}c = ۰ \quad (\frac{۱}{\sqrt{۲}})^{\frac{c}{a}} = ? \quad b+c = ۲a \Rightarrow c = ۲a-b$ —۱۰

$\alpha+\beta = \frac{-b}{a} \Rightarrow \frac{a}{-b} = \log^{۴} \Rightarrow \frac{-۴a}{-b} = \log^{۴} \Rightarrow \frac{۴a}{b} = ۲\log^{۲}$

$\Rightarrow \frac{۲a}{b} = \log^{۲} \Rightarrow \frac{۲a}{\log^{۲}} = b \Rightarrow \frac{c}{a} = \frac{۲a-b}{a} = ۲ - \frac{b}{a} = ۲ - (\frac{۲a}{\log^{۲}} \times \frac{۱}{a})$

$\Rightarrow \frac{c}{a} = ۲ - \frac{۲}{\log^{۲}} = ۲ - ۲\log_{۲}^{۱۰} \rightsquigarrow (\frac{۱}{\sqrt{۲}})^{\frac{c}{a}} = (۲^{-\frac{۱}{۸}})^{۲-۲\log_{۲}^{۱۰}}$

$\frac{۲}{\log^{۲}} = ۲\log_{۲}^{۱۰} \quad \Rightarrow ۲^{-\frac{۱}{۴}+\frac{۱}{۴}\log_{۲}^{۱۰}} = ۲^{-\frac{۱}{۴}} \times (۲^{\frac{۱}{۴}(\log_{۲}^{۵}+\log_{۲}^{۲})}) = ۲^{\frac{۱}{۴}\log_{۲}^{۵}} = ۵^{\frac{۱}{۴}} = \sqrt[۴]{۵}$