نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ۲۳ ..... کلاس ..... ۱۱ A .....

۱

$\log_n^m = a \Rightarrow \frac{\log m}{\log n} = a$ ، $\log_{mn}^{m^2 n} = \frac{2\log m + \log n}{\log m + \log n} = \frac{2a+1}{a+1} = b$

$\Rightarrow \frac{a+1+a}{a+1} = b \Rightarrow 1 + \frac{a}{a+1} = b \Rightarrow [b] = 1$

۲

الف)

$\frac{x}{\log_{\frac{1}{2}} x} \geq 0 \Rightarrow x \geq 0 \ , \ x > 0 \ , \ \log_2 x \neq 0 \Rightarrow x \neq 1$

	0		1	
	−	+	−	

$\Rightarrow (0,1)$

ب)

$x^2 - x - 2 > 0 \rightarrow \Delta = 9 \rightarrow x = -1, 2$

	−1		2	
	+	−	+	

$\Rightarrow (-\infty, -1) \cup (2, +\infty)$ ①

$x^2 - 1 \geq 0 \Rightarrow x^2 \geq 1 \Rightarrow (-\infty, -1] \cup [1, +\infty)$ ②

$\sqrt{x^2 - 1} + 1 \neq 0$ همیشه صفر می‌شود $\Rightarrow D = ① \cap ② = (-\infty, -1) \cup (2, +\infty)$

۳

$2\log_{\frac{1}{3^2}}^{a} + \log_a^{3} = 2 \Rightarrow \log_{3^2}^{a} + \log_a^{3} = 2 \Rightarrow \frac{1}{\log_a^3} + \log_a^3 = 2$

$\xrightarrow{\log_a^3 = b} \frac{1}{b} + b = 2 \Rightarrow b^2 + 1 = 2b \Rightarrow b^2 - 2b + 1 = 0 \Rightarrow b = 1$

$\log_a^3 = 1 \Rightarrow a = 3$

۴

$(\log 5 - \log 3)x^2 + (2\log 3)x - \log 3 - \log 5 = 0$

مجموع ضرایب صفرات $\Rightarrow x \rightarrow 1$

$\frac{c}{a} = \frac{-\log 3 - \log 5}{\log 5 - \log 3} = \frac{\log 3 + \log 5}{\log 3 - \log 5} = \frac{\log 3 + (\log 10 - \log 2)}{\log 3 - (\log 10 - \log 2)} = \frac{0/4 + 1 - 0/3}{0/4 - 1 + 0/3}$

$= \frac{1/1}{-0/3} = \frac{-11}{3}$ اختلاف: $\frac{-11}{3} - 1 = \frac{-14}{3}$

۵

$\log_{14}^{10} = \frac{\log_2^{10}}{\log_2^{14}} = \frac{\log_2^5 + \log_2^2}{\log_2^7 + \log_2^2} = \frac{\log_2^5 + 1}{2/8 + 1} \xrightarrow{(1.2)^5 = \frac{1}{1.25} = 2} \frac{2+1}{3/8} = \frac{3}{3/8} \simeq 0/71$

$\log_{15}^{6} = \dfrac{1}{\log_{6}^{15}} = \dfrac{\log_{3}^{6}}{\log_{3}^{15}} = \dfrac{\log_{3}^{2} + 1}{\log_{3}^{5} + 1} = \dfrac{\frac{10}{16} + 1}{2.5} = \dfrac{\frac{26}{16}}{2.5} = 0.65$ ۶

$\log_{2^3}^{18} = \frac{1}{3}\log_{2}^{18} = m \Rightarrow \frac{1}{3}(\log_{2}^{9} + \log_{2}^{2}) = m \Rightarrow \frac{2}{3}(\log_{2}^{3}) + \frac{2}{3} = m$

$\Rightarrow \frac{2}{3}\log_{2}^{3} = m - \frac{2}{3} \Rightarrow \log_{2}^{3} = \frac{3}{2}m - 1$

$\log_{4}^{12} = \frac{1}{2}\log_{2}^{12} = \frac{1}{2}(\log_{2}^{4} + \log_{2}^{3}) = \frac{1}{2}(2 + \log_{2}^{3}) = \frac{1}{2}(2 + \frac{3}{2}m - 1)$ ۷

$\Rightarrow \frac{1}{2}(\frac{3}{2}m + 1) = \frac{3}{4}m + \frac{1}{2}$

$(0.4)^{2x-1} = (\frac{5}{2})^{3x^2} \Rightarrow (\frac{2}{5})^{2x-1} = (\frac{5}{2})^{3x^2} \Rightarrow 3x^2 = 1 - 2x \Rightarrow 3x^2 + 2x - 1 = 0$

$\Rightarrow \Delta = 4 + 4(3) = 16 \Rightarrow x = \dfrac{-2 \pm 4}{6} \nearrow -1, \searrow \frac{1}{3} \Rightarrow 9x + 1 > 0 \Rightarrow$ غ ق ق -1 ۸

$\log_{8}^{(9x+1)} = \log_{8}^{4} = \log_{2^3}^{2^2} = \boxed{\frac{2}{3}}$

$\log_{2^3}^{b} = \frac{1}{3}\log_{2}^{b} = \frac{2}{3} + \frac{2}{3}a \Rightarrow \log_{2}^{b} = 2 + 2a$ ، $\log_{2}^{3} = a \Rightarrow 3 = 2^a$

$\log_{2}^{b} = 2 + 2a \Rightarrow (2)^{2+2a} = b \Rightarrow 2^2 \times 2^{2a} = b \Rightarrow 4 \times \underbrace{(2^a)^2}_{9} = b \Rightarrow b = 36$ ۹

$\log_{10}^{(3b-8)} = \log_{10}^{100} = \boxed{2}$

راه حل: $\dfrac{-b}{a} = \dfrac{+b}{+4a} \xrightarrow{\text{عکس}} \dfrac{4a}{b} = \log 4 \Rightarrow \underbrace{\dfrac{2a}{b} = \log 2}_{\frac{a}{b} = \frac{\log 2}{2}}$ / $a = \dfrac{b+c}{2} \Rightarrow 2a - b = c$

$\dfrac{c}{a} = \dfrac{2a-b}{a} = 2 - \dfrac{b}{a} = 2 - \dfrac{2}{\log 2} \Rightarrow (2^{-\frac{1}{8}})^{\frac{c}{a}} = (2^{-\frac{1}{8}})^{2 - \frac{2}{\log 2}} = 2^{-\frac{1}{4} + \frac{1}{4\log 2}} = 2^{-\frac{1}{4}} \times 2^{\frac{1}{4\log 2}}$ ۱۰

(یادآوری: $\log 2 \times \log_{2}^{10} = 1 \Rightarrow \log_{2}^{10} = \dfrac{1}{\log 2} \Rightarrow (2)^{\frac{1}{\log 2}} = 10$)

$\Rightarrow 2^{-\frac{1}{4}} \times (\underbrace{2^{\frac{1}{\log 2}}}_{10})^{\frac{1}{4}} = \dfrac{1}{\sqrt[4]{2}} \times \sqrt[4]{10} = \dfrac{\sqrt[4]{10}}{\sqrt[4]{2}} = \sqrt[4]{5}$