نام و نام خانوادگی ...... حسین الکریمی خواه ...... پاسخنامه تشریحی تکلیف شماره ۲۳ ..... کلاس یازدهم ...(۸۱).........

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$$\log_n m = a > 0 \quad \begin{cases} n>1 \to m>1 \\ 0<n<1 \to 0<m<1 \end{cases}$$

$$\log_{mn} m^2 n = b \qquad b = \log_{mn} m^2 n = \log_{mn} m^2 + \log_{mn} n = \frac{2\log_n m}{\log_n m + \log_n n} + \frac{\log_n n}{\log_n m + \log_n n} = \frac{2a+1}{a+1}$$

$$[b] = ? = \left[\frac{2a+1}{a+1}\right] = \left[\frac{a}{a+1} + \frac{a+1}{a+1}\right] = \left[\frac{a}{a+1} + 1\right] = \left[\frac{a}{a+1}\right] + 1 \xrightarrow{a>0} = 0 + 1 = 1$$

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الف)

$$y = \sqrt{\frac{x}{\log_{\frac12} x}} \to \frac{x}{\log_{\frac12} x} \geqslant 0 , \quad \log_{\frac12} x \neq 0 \to x \neq 1 \qquad D_f : (0,1)$$

$$\text{طبق دامنه } \log \to x > 0 \Rightarrow \log_{\frac12} x > 0 \Rightarrow x < \left(\tfrac12\right)^0 \Rightarrow x < 1$$

ب)

$$y = \frac{\log_4 (x^2 - x - 2)}{\sqrt{x^2-1} + 1}$$

$$x^2 - x - 2 > 0 \Rightarrow x < -1 , \; x > 2 \quad (1)$$

$$x^2 - 1 \geqslant 0 \to x^2 \geqslant 1 \to x \geqslant 1 , \; x \leqslant -1 \quad (2)$$

$$(1) \cap (2) \to x < -1 , \; x > 2$$

$$D_f : (-\infty , 1) \cup (2 , +\infty)$$

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$$2\log_x a + \log_a \sqrt{x} = 2 \xrightarrow[\text{لگاریتم}]{\text{ویژگی‌های}} 2\log_x a + \log_{\sqrt{x}} a = 2 \xrightarrow[\text{لگاریتم}]{\text{ویژگی‌های}} 2\log_x a + 2\log_x a = 2$$

$$4\log_x a = 2 \to \log_x a = \frac12 \xrightarrow{x=9} \log_9 a = \frac12 \to a = \sqrt{9} = \pm 3 \xrightarrow{\text{دامنه } \log} a = +3$$

$$1 + \frac{11}{3} = \frac{14}{3} \simeq 4.6$$

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$$\left[\log \frac{5}{3}\right] x^2 + \left[\log 9\right] x - \log 15 = 0$$

$$|x_1 - x_2| = \left|1 - \left(\cancel{-\frac{1.1}{0.3}}\right)\right| \simeq |-2.7| = 2.7$$

۲٫۷ واحد اختلاف دارند.

$$\text{مجموع ضرایب}: \; \log\frac53 + \log 9 - \log 15 = \log \frac{\frac53 \times 9}{15} = \log 1 = 0 \to \begin{cases} x_1 = 1 \\ x_2 = \dfrac{-\log 15}{\log \frac53} \end{cases}$$

$$x_2 = \frac{-[\log 5 + \log 3]}{\log 5 - \log 3} = \frac{-[\log 10 - \log 2 + \log 3]}{\log 10 - \log 2 - \log 3} = \frac{-(1 - 0.3 + 0.4)}{1 - 0.3 - 0.4} = \frac{-1.1}{0.3}$$

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$$\log_{14} 10 = \log_7 2 + \log_7 5 = \frac{1}{\log_2 7} + \frac{\log_2 5}{\log_2 7} = \frac{1}{\log_2 7} + \frac{\frac{1}{\log_5 2}}{\log_2 7} = \frac{1 + \frac{1}{\log_5 2}}{\log_2 7}$$

$$\begin{matrix} \log_2 7 = 2.8 \\ \log_5 2 = 0.5 \end{matrix} \longrightarrow \frac{1 + \frac{1}{0.5}}{2.8 + 1} = \cancel{\frac{3}{2.8} \simeq 1.07} \qquad \frac{15}{19}$$

$$\frac{\log_2 5 + \log_2 2}{\log_2 7 + \log_2 2}$$

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$\log_{15}^{6} = \log_{15}^{2} + \log_{15}^{3} = \frac{1}{\log_{2}^{15}} + \frac{1}{\log_{3}^{15}} = \frac{1}{\log_{2}^{5} + \log_{2}^{3}} + \frac{1}{\log_{3}^{5} + \log_{3}^{3}}$ ۶

$\log_{15}^{6} = \frac{1}{\log_{2}^{3} + \frac{\log_{3}^{5}}{\log_{3}^{2}}} + \frac{1}{\log_{3}^{5} + 1} = \frac{1}{\log_{2}^{3} + \frac{\log_{3}^{5}}{\frac{1}{\log_{2}^{3}}}} + \frac{1}{\log_{3}^{5} + 1} \xrightarrow[\log_{2}^{3} = 1.6]{\log_{3}^{5} = 1.5}$

$= \frac{1}{1.6 + 1.5 \times 1.6} + \frac{1}{1.5 + 1} = \frac{1}{4} + \frac{1}{2.5} = \frac{6.5}{10} \longrightarrow \frac{65}{100}$

$\log_{8}^{18} = m \qquad \log_{4}^{12} = ? = \frac{1}{2}\log_{2}^{12} = \frac{1}{2}\left(\log_{2}^{3} + \log_{2}^{2} + \log_{2}^{2}\right)$ ۷

$\log_{2^3}^{18} = m \qquad \log_{4}^{12} = \frac{1}{2}\left(\log_{2}^{3} + 2\right) \xrightarrow{\log_{2}^{3} = \frac{3m-1}{2}} \frac{1}{2}\left(\frac{3m-1}{2} + 2\right) = \frac{3}{4}(m+1)$

$\frac{1}{3}\log_{2}^{18} = m \rightarrow \frac{1}{3}\left(\log_{2}^{9} + \log_{2}^{2}\right) = m$

$\frac{1}{3}\left(2\log_{2}^{3} + 1\right) = m \Longrightarrow 2\log_{2}^{3} + 1 = 3m \Longrightarrow \log_{2}^{3} = \frac{3m-1}{2}$

$\left(\frac{4}{10}\right)^{2x-1} = \left(\frac{125}{8}\right)^{x^2} \qquad \log_{8}^{(9x+1)} = \begin{cases} \log_{8}^{(9(-1)+1)} : \text{غ ق ق} : (9(-1)+1) < 0 \\ \log_{8}^{(9(\frac{1}{3})+1)} = \log_{8}^{4} = \log_{2^3}^{2^2} = \boxed{\frac{2}{3}} \end{cases}$ ۸

$\left(\frac{4}{10}\right)^{2x-1} = \left(\left(\frac{5}{2}\right)^{3}\right)^{x^2}$

$\xrightarrow{\frac{4}{10} = \frac{1}{\frac{5}{2}}} \left(\frac{5}{2}\right)^{-2x+1} = \left(\frac{5}{2}\right)^{3x^2} \Longrightarrow 3x^2 = -2x + 1 \Longrightarrow 3x^2 + 2x - 1 = 0 \begin{cases} x = -1 \\ x = \frac{1}{3} \end{cases}$

$\log_{2}^{3} = a \qquad \log^{(3b-8)} = \log^{(108-8)} = \log^{100}$ ۹

$\log_{8}^{b} = \frac{2}{3}\left(1 + \log_{2}^{3}\right) = \frac{2}{3} + \frac{2}{3}\log_{2}^{3} = \frac{2}{3} + \log_{8}^{9} = \log_{8}^{b} \qquad \log^{100} = \boxed{2}$

$\log_{8}^{b} = \frac{2}{3}\log_{2}^{2} + \log_{8}^{9} = \log_{8}^{4} + \log_{8}^{9} = \log_{8}^{36} \Longrightarrow b = 36$

$\left[\frac{1}{\sqrt[8]{2}}\right]^{\frac{c}{a}} = \left[2^{(\frac{-1}{8})}\right]^{\frac{c}{a}} = 2^{\frac{-c}{8a}} \;/\; \log^{4} = \frac{1}{\alpha + \beta} = \frac{1}{S} = \frac{1}{\frac{-b}{a}} = \frac{a}{-b} \xrightarrow{2a = b + c} \frac{a}{-b} = \frac{-4a}{c - 2a}$ ۱۰

$\log^{4} = \frac{-4a}{c - 2a} \longrightarrow \log_{4}^{10} = \frac{c - 2a}{-4a} = \frac{c}{-4a} + \frac{1}{2} \xrightarrow{\times \frac{1}{2}} \log_{16}^{10} = \frac{c}{-8a} + \frac{1}{4}$

$\longrightarrow \frac{-c}{8a} = \log_{16}^{10} - \frac{1}{4} = \frac{1}{4}\log_{2}^{10} - \frac{1}{4} = \log_{2}^{\sqrt[4]{10}} - \log_{2}^{\sqrt[4]{2}} = \log_{2}^{\frac{\sqrt[4]{10}}{\sqrt[4]{2}}}$

$2^{\frac{-c}{8a}} = 2^{\log_{2}^{\frac{\sqrt[4]{10}}{\sqrt[4]{2}}}} = \frac{\sqrt[4]{10}}{\sqrt[4]{2}} = \sqrt[4]{5}$