سارا سامانی - یازدهم دختر A - تکلیف شماره ۲۳

۱-

$\log_n m = a$

$\log_{mn} m^2 n = b \Rightarrow \dfrac{\log_n m^2 n}{\log_n mn} = \dfrac{\log_n m^2 + \cancel{\log_n n}^{\,1}}{\log_n m + \cancel{\log_n n}_{\,1}} = \dfrac{2\log_n m + 1}{\log_n m + 1} = b \Rightarrow \dfrac{2a+1}{a+1} = b \Rightarrow$

$1 + \dfrac{a}{a+1} = b \xrightarrow[\text{از دو طرف}]{\text{براکت}} \left[1 + \dfrac{a}{a+1}\right] = [b] \Rightarrow 1 + \underbrace{\left[\dfrac{a}{a+1}\right]}_{0} = [b] \Rightarrow [b] = 1$

۲-

الف)

$y = \sqrt{\dfrac{x}{\log_{\frac{1}{2}} x}} \to x > 0 \quad \text{I}$ $\qquad \dfrac{x}{\log_{\frac{1}{2}} x} \geq 0$ $\qquad \begin{array}{ccccc} & 0 & & 1 & \\ - & \bullet & + & \circ & - \end{array}$ $\qquad x \in (0,1) \ \text{II} \xrightarrow[\text{I} \cap \text{II}]{} D_f = (0,1)$

$\log_{\frac{1}{2}} x = 0 \Rightarrow x = 1$

ب)

$y = \dfrac{\log_4 (x^2 - x - 2)}{\sqrt{x^2 - 1} + 1} \Rightarrow x^2 - x - 2 > 0 \Rightarrow (x-2)(x+1) > 0$ $\qquad \begin{array}{ccccc} & -1 & & 2 & \\ + & \circ & - & \circ & + \end{array}$ $\qquad x \in (-\infty, -1) \cup (2, +\infty) \ \text{I}$

$\sqrt{x^2-1} + 1 \neq 0 \Rightarrow \sqrt{x^2-1} \neq -1$ همواره برقرار $\qquad \sqrt{x^2-1} \Rightarrow x^2 - 1 \geq 0 \quad x^2 \geq 1 \Rightarrow x \in (-\infty, -1] \cup [1, +\infty) \ \text{II}$

$\xrightarrow[\text{I} \cap \text{II}]{} D_f = (-\infty, -1) \cup (2, +\infty)$

۳-

$2\log_x a + \log_a \sqrt{x} = 2 \xrightarrow{x=9} \underbrace{\cancel{2\log_9 a}}_{\underbrace{2\log_{3^2} a}_{\log_3 a}} + \underbrace{\cancel{\log_a \sqrt{9}}}_{\log_a 3} = 2 \Rightarrow \log_3 a + \log_a 3 = 2 \Rightarrow \dfrac{1}{\log_a 3} + \log_a 3 = 2, \ \log_a 3 = A$

$\dfrac{1}{A} + A = 2 \Rightarrow \dfrac{A^2 + A}{A} = 2 \Rightarrow A^2 - A = 0 \begin{cases} A = 0 \to \log_a 3 = 0 & \text{غیرممکن} \\ A = 1 \quad \log_a 3 = 1 \Rightarrow a = 3 \end{cases}$

۴-

$\log 3 \simeq 0.4$

$\log 2 \simeq 0.3$

$(\log \frac{5}{3}) x^2 + (\log 9) x - \log 15 = 0 \Rightarrow \log \frac{5}{3} = \log 5 - \underbrace{\cancel{\log 3}}_{0.4} = \log \frac{10}{2} - 0.4 = \underbrace{(\cancel{\log 10} - \cancel{\log 2})}_{1 - 0.3 = 0.7} - 0.4$

$= 0.3$

$\log 9 = \log 3^2 = 2\log 3 = 0.8$

$\log 15 = \log 3\times 5 = \log 3 + \log \frac{10}{2} = \underset{0.4}{\cancel{\log 3}} + (\underset{1}{\cancel{\log 10}} - \underset{-0.3 = 0.7}{\cancel{\log 2}}) = 1.1$

$y = 0.3x^2 + 0.8x - 1.1 = 0 \qquad \Delta = 1.96 \qquad |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{1.4}{0.3} = \frac{14}{3}$

۵-

$\log_2 7 \simeq 2.8$

$\log_5 2 = 0.5$

$\log_{14} 10 = ?$

$\log_2 5 = \frac{1}{\log_5 2} = \frac{1}{0.5} = 2$

$\log_{14} 10 = \frac{\log_2 10}{\log_2 14} = \frac{\overset{2}{\cancel{\log_2 5}} + \overset{1}{\cancel{\log_2 2}}}{\underset{2.8}{\cancel{\log_2 7}} + \underset{1}{\cancel{\log_2 2}}} = \frac{3}{3.8} = \frac{15}{19}$

۶-

$\log_3 5 \simeq 1.5$

$\log_2 3 = 1.6$

$\log_{15} 6 = ?$

$\log_3 2 = \frac{1}{\log_2 3} = \frac{1}{1.6} = 0.625$

$\log_{15} 6 = \frac{\log_3 6}{\log_3 15} = \frac{\overset{0.625}{\cancel{\log_3 2}} + \overset{1}{\cancel{\log_3 3}}}{\underset{1.5}{\cancel{\log_3 5}} + \underset{1}{\cancel{\log_3 3}}} = \frac{1.625}{1.5}$

۷-

$\log_8 18 = m \Rightarrow \log_{2^3} 18 = m \Rightarrow \frac{1}{3}\log_2 18 = m \Rightarrow \log_2 18 = 3m \Rightarrow \log_2 18 = 3m \Rightarrow \log_2 9 + \underset{1}{\cancel{\log_2 2}} = 3m$

$\log_2 3^2 = 3m - 1 \Rightarrow 2\log_2 3 = 3m - 1 \Rightarrow \log_2 3 = \frac{3m-1}{2}$

$\log_4 12 = \frac{\log_2 12}{\underset{2}{\cancel{\log_2 4}}} = \frac{\log_2 6 + \cancel{\log_2 2}}{2} = \frac{\overset{1}{\cancel{\log_2 2}} + \overset{\frac{3m-1}{2}}{\cancel{\log_2 3}} + 1}{2} = \frac{\frac{3m-1}{2} + 2}{2} = \frac{3m+3}{4}$

۸-

$\left(\frac{4}{10}\right)^{2x-1} = \left(\frac{5^3}{2^3}\right)^{x^2} \Rightarrow \left(\frac{2}{5}\right)^{2x-1} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow \left(\frac{5}{2}\right)^{1-2x} = \left(\frac{5}{2}\right)^{3x^2} \Rightarrow 3x^2 = 1 - 2x \Rightarrow$

$3x^2 + 2x - 1 = 0 \qquad \Delta = 16 \qquad \frac{-2 \pm 4}{6} \begin{cases} \nearrow -1 \text{ غ ق ق} \\ \searrow \frac{1}{3} \end{cases}$

$\log_8 (9x+1) = \log_8 4 = \log_{2^3} 2^2 = \frac{2}{3}\underset{1}{\cancel{\log_2 2}} = \frac{2}{3}$

9-

$\log_2^3 = a$

$\log_8^b = \frac{2}{3}(1+a) \Rightarrow \log_{2^3}^b = \frac{2}{3}(1+a) \Rightarrow \frac{1}{\cancel{3}}\log_2^b = \frac{2}{\cancel{3}}(1+a) \Rightarrow \log_2^b = 2(1+a) \Rightarrow$

$\log_2^b = \underbrace{2}_{\log_2^4} + 2\log_2^3 \Rightarrow \log_2^b = \log_2^4 + \log_2^9 \Rightarrow \log_2^b = \log_2^{36} \Rightarrow b = 36$

$\log(3b-8) = \log(108-8) = \log 100 = \boxed{2}$

10-

$\frac{1}{5} = \frac{a}{b} \Rightarrow \frac{4a}{b} = \log 4 \Rightarrow \frac{\cancel{4}^{2}a}{b} = \cancel{2}\log 2 \Rightarrow \frac{2a}{b} = \log 2 \Rightarrow \frac{a}{b} = \frac{\log 2}{2}$ (1)

$\frac{b+c}{2} = a \Rightarrow c = 2a-b$

$\frac{c}{a} = \frac{2a-b}{a} = 2 - \frac{b}{a} \overset{(1)}{\Rightarrow} 2 - \frac{2}{\log 2} = 2\left(1 - \underbrace{\frac{1}{\log 2}}_{\log_2^{10}}\right) = 2 - 2\log_2^{10}$

$\left(\underbrace{\frac{1}{\sqrt[8]{2}}}_{2^{-\frac{1}{8}}}\right)^{2-2\log_2^{10}} \Rightarrow \left(2^{-\frac{1}{8}}\right)^{2-2\log_2^{10}} = \left(2^{-\frac{1}{8}}\right)^{2(1-\log_2^{10})} = 2^{-\frac{1}{4}(1-\log_2^{10})} =$

$2^{-\frac{1}{4}(\log_2^2 - \log_2^{10})} = 2^{-\frac{1}{4}(\log_2^{0.2})} = 2^{(\log_{16}^5)} = 2^{(\frac{1}{4}\log_2^5)} = 5^{(\frac{1}{4}\log_2^2)} =$

$5^{\frac{1}{4}} = \boxed{\sqrt[4]{5}}$