آرشا یاری   یازدهم دختر A   تکلیف شماره ۲

۱- $2x^2+y^2-4x+6y+K=0 \longrightarrow y^2+6y+\underbrace{(2x^2-4x+K)}_{c}=0$

$\Delta=b^2-4ac=36-8x^2+16x-4K \rightsquigarrow -8x^2+16x=-8\left((x-1)^2-1\right)=-8(x-1)^2+8$

$\Delta=44-4K-8(x-1)^2\leq 44-4K \longrightarrow 0\geq 44-4K \to K\geq 11$ $\xrightarrow{\text{ناشی}}$ $\boxed{K=11}$

۲- $a^2+4a=3a+2 \to a^2+a-2=0 \to$ $\boxed{x=1 \text{ ق ق}}$ $x=-2$ غ ق ق $a>0$

$$f(x)=\begin{cases} x^2+4x & ; x\geq 1 \\ \text{-----} & \\ 2x+3 & ; x\leq 1 \end{cases} \Longrightarrow f(1)=1+4=5$$

۳- $$f(x)=\begin{cases} 2x^2-b & ; |x+1|\geq 2 \\ a^2+2x & ; -3<x\leq -1 \end{cases}$$

$2(-3)^2-b=a+2(-3)$

$18-b=a-6 \longrightarrow a+b=18+6=24$

۴- $6x-a\geq 0 \longrightarrow x\geq \frac{a}{6}$ $b-3x\geq 0$ $x\leq \frac{b}{3}$

$\Longrightarrow D_f=\left[\frac{a}{6},\frac{b}{3}\right]=\{2\} \longrightarrow \frac{a}{6}=\frac{b}{3}=2 \longrightarrow a=12 \quad b=6 \quad \frac{b}{a}=\frac{6}{12}=\frac{1}{2}$

۵- الف) $\frac{x+1}{|x-3|}\geq 0$

x		-1		3	
	$-$	0	$+$	$\nexists$	$+$

$D=[-1,3)\cup(3,+\infty)$ (I)

$\frac{6-x}{x^2+2x+4}\geq 0$

x		6	
	$+$	0	$-$

$D=(-\infty,6]$ (II) $(I)\cap(II)=[-1,3)\cup(3,6]$

ب) $[x]-4\geq 0 \to [x]\geq 4 \longrightarrow x\geq 4$ $D=[4,+\infty)$ (I)

$2-[x]\geq 0 \longrightarrow [x]\leq 2 \to x<3$ $D=(-\infty,3)$ (II)

$(I)\cap(II)=\varnothing$

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$t = x^2$

۶- الف) $x^4 - 13x^2 + 36 = t^2 - 13t + 36 = 0 \quad t_1 = 9 \to x_1 = \pm 3 \quad t_2 = 4 \to x_2 = \pm 2$

$x^2 - x - 6 \to x = -2 \quad x = 3$

$\dfrac{x^2 - x - 6}{x^4 - 13x^2 + 36} \geq 0$

x		-3		-2^*		2		3^*	
	$+$		$-$		$-$		$+$		$+$

$D = (-\infty, -3) \cup (2, 3) \cup (3, +\infty)$

ب) $\dfrac{x^3 - 2x^2 - 5x + 6}{x^3 + 2x^2 - 5x - 6} \geq 0 \to \dfrac{(x-1)(x-3)(x+2)}{(x+1)(x+3)(x-2)} \geq 0$

x		-3		-2		-1		1		2		3	
	$+$		$-$		$+$		$-$		$+$		$-$		$+$

$D = (-\infty, -3) \cup [-2, -1) \cup [1, 2) \cup [3, +\infty)$

۷- الف) $[-x] - 2 \geq 0 \to [-x] \geq 2 \to x \leq -2 \to D = (-\infty, -2]$

ب) $[x]^2 - 2[x] - 3 \xrightarrow{[x] = n} n^2 - 2n - 3 \neq 0 \to (n-3)(n+1) \neq 0 \quad n \neq 3 \quad n \neq -1$

$[x] \neq 3 \Rightarrow (-\infty, 3) \cup [4, +\infty) \ (1) \quad [x] \neq -1 \Rightarrow D = (-\infty, -1] \cup (0, +\infty) \ (2)$

$(1) \cap (2) = (-\infty, -1] \cup (0, 3) \cup [4, +\infty)$ دامنه صورت IR بوده تاثیری ندارد.

۸- الف) $\to \begin{cases} \sin x \neq 0 & x \neq K\pi \\ \cos x \neq 0 & x \neq K\pi + \frac{\pi}{2} \\ \tan x + 1 \neq 0 \to \tan x \neq -1 & x \neq K\pi + \frac{3\pi}{4} \end{cases} \Rightarrow D = R - \{K\pi, K\pi + \frac{\pi}{2}, K\pi + \frac{3\pi}{4}\}$

ب) $1 - 4\sin^2 x \geq 0 \to 4\sin^2 x \leq 1 \to \sin^2 x \leq \frac{1}{4} \quad (1)$

ج.م $= (1) \cap (2) = [0, \frac{1}{4}]$

$0 \leq \sin^2 x \leq 1 \quad (2)$

۹- الف) $\begin{cases} x - 1 > 0 \to x > 1 & (1) \\ 1 - \log_{\frac{1}{2}}^{x-1} \geq 0 \to \log_{\frac{1}{2}}^{x-1} \leq 1 \to x - 1 \leq \frac{1}{2} \to x \leq \frac{3}{2} & (2) \end{cases}$

$(1) \cap (2) = (1, \frac{3}{2}]$

آروشا یاوری

(1)

۹- الف) $x^2 - x \geq 0 \rightarrow x(x-1) \geq 0$

x		0		1	
	$+$	0	$-$	0	$+$

$D = (-\infty, 0] \cup [1, +\infty)$

(2)

$x^2 - 3x > 0 \rightarrow x(x-3) > 0$

x		0		3	
	$+$	0	$-$	0	$+$

$D = (-\infty, 0) \cup (3, +\infty)$

(3)

$1 - \log_4^{x^2-3x} \neq 0 \rightarrow \log_4^{x^2-3x} \neq 1 \rightarrow x^2 - 3x \neq 4 \rightarrow x^2 - 3x - 4 \neq 0 \rightarrow D = \mathbb{R} - \{-4, 1\}$

$(1) \cap (2) \cap (3) = (-\infty, -4) \cup (-4, 0) \cup (3, +\infty)$

۱۰- الف) $2^{4x - x^2} - 8 \geq 0 \rightarrow 2^{4x - x^2} \geq 2^3 \rightarrow 4x - x^2 \geq 3$

$-x^2 + 4x - 3 \geq 0 \rightarrow -(x^2 - 4x + 3) \geq 0$

x		-3		-1	
	$-$	0	$+$	0	$-$

$D = [-3, -1]$

ب) $\dfrac{3x+5}{2x+4} \in W \rightarrow 3x + 5 \in 2Wx + 4W \rightarrow 3x - 2Wx \in 4W - 5 \rightarrow x \in \dfrac{4W - 5}{3 - 2W}$

$\rightarrow \left\{ x \mid x = \dfrac{4K - 5}{3 - 2K}, K \in W \right\}$