نام و نام خانوادگی ..... پاسخنامه تشریحی تکلیف شماره ..2.. کلاس .....A.....

۱

$2x^2+y^2-4x+4y+k=0$

$\rightarrow 2x^2-4x = 2\underbrace{(x^2-2x)}_{(x-1)^2-1} \rightarrow 2[(x-1)^2-1] \rightarrow 2(x-1)^2-2$

$\rightarrow y^2+4y = (y+2)^2-4$ در حاصل قرار می‌دهیم $\rightarrow 2(x-1)^2-2+(y+2)^2-4+k=0$

$\rightarrow 2(x-1)^2+(y+2)^2 = 6-k \xrightarrow{6-k=0} k=6$

۲

$f(x)=\begin{cases} x^2+4x & x\geq -2 \\ 2x+3+2 & x<-2 \end{cases} \rightarrow x^2+4x = 2x+3+2 \xrightarrow{f(1)} 1+4 = 2+2+a$

$a = 4+\frac{a}{1}$

$\rightarrow \begin{cases} 1+4=a \\ 2+1+2=a \end{cases} \rightarrow f(1)=a$

۳

$F(x)=\begin{cases} 2x^2-b & |x+1|\geq 2 \\ a+2x & -3\leq x\leq -1 \end{cases}$ به جای x "-3" قرار می‌دهیم $\rightarrow 2x^2-b = a+2x \rightarrow 18-b = a+(-6)$

$\rightarrow a+b = 18+6 \rightarrow a+b=24$

۴

$y=\sqrt{4x-8}+\sqrt{b-3x}$

$4x-8\geq 0 \rightarrow 4x\geq 8$

$b-3x\geq \rightarrow -3x\geq -b \rightarrow 3x\leq b$

$\rightarrow \frac{b}{8} = \frac{3x}{4x} = \frac{1}{2}$

۵

الف) $\sqrt{\frac{x+1}{|x-3|}}+\sqrt{\frac{4-x}{x^2+2x+4}} \rightarrow \frac{x+1}{|x-3|}\geq 0 \rightarrow \begin{matrix} x\geq -1 \\ x\neq 3 \end{matrix}$ (۱)

x^2+2x+4 دلتا $\rightarrow b^2-4ac$ (۲)

$4-12<0 \rightarrow$ همیشه مثبت

$4-x\geq 0 \rightarrow x\leq 4$

$\rightarrow -1\leq x\leq 4 \; , \; x\neq 3 \rightarrow [-1,3)\cup(3,4]$

ب) $\sqrt{[x]-4}+\sqrt{2-[x]}$ $\rightarrow$ به ازای هیچ مقدار x تابع نداریم

$[x]-4\geq 0 \rightarrow [x]\geq 4$

$2-[x]\geq 0 \rightarrow [x]\leq 2$

$\frac{(x-3)(x+2)}{(x-3)(x+3)(x+2)(x-2)} \to \frac{1}{(x+3)(x-2)} \to x \neq 3, \pm 2 \to$ (sign table) $\to (-\infty,-3) \cup (+2,+\infty) - \{3\}$

6

الف) $\sqrt{\frac{x^2-x-6}{x^4-13x^2+36}} \to \frac{x^2-x-6}{x^4-13x^2+36} \geq 0$

$x^2-x-6 \to (x-3)(x+2) \to x = 3, -2$ ادامه

$\xrightarrow{x^2=t} t^2-13t+36 \to (t-9)(t-4)=0 \to x^2=9 \to x=\pm 3$ ، $x^2=4 \to x=\pm 2$

(score: 2)

ب) $\sqrt{\frac{x^3-2x^2-5x+6}{x^3+2x^2-5x-6}}$ $\xrightarrow[\div(x-2)]{\div(x-1)} \frac{(x-1)(x-3)(x+2)}{(x-2)(x+1)(x+3)} \to$ (sign table: $-3, -2, -1, 1, 2, 3$) $\to (-\infty,-3) \cup [-2,-1) \cup [1,2) \cup [3,+\infty)$

7

الف) $\sqrt{[-x]-2} \to [-x]-2 \geq 0 \to [-x] \geq 2 \to -x \geq 2 \to x \leq -2 \to (-\infty,-2]$

(score: 1.5)

$R - [3,4) \cup [-1,0)$

ب) $\frac{5x^2+6}{[x]^2-2[x]-3} \neq 0 \xrightarrow{[x]=n} n^2-2n-3=0 \to (n-3)(n+1)=0 \to n=3 \to [x]=3 \to 3 \leq x < 4$

$n=-1 \to [x]=-1 \to -1 \leq x < 0$

8

الف) $\frac{\cot x+1}{\tan x+1}$ ($\cot = \frac{\cos}{\sin}$, $\tan = \frac{\sin}{\cos}$) $\to \sin \neq 0 \to x \neq k\pi \mid k \in Z$

$\tan x+1 \to \tan x = -1 \to x = -\frac{\pi}{4} + k\pi \mid k \in Z$

$\to \{x \in R \mid \frac{k\pi}{2}, -\frac{\pi}{4}+k\pi \mid k \in Z\}$

(score: 2)

ادامه $\to R - \{k\pi \pm \frac{\pi}{6}\}$

ب) $\sqrt{1-4\sin^2 x} \to 1-4\sin^2 x \geq 0 \to -4\sin^2 x \geq -1 \to \sin^2 \leq \frac{1}{4} \to -\frac{1}{2} \leq \sin x \leq +\frac{1}{2}$

نقاط:

$\sin x = \frac{1}{2} \to x = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi \mid k \in Z$

$\sin x = -\frac{1}{2} \to x = -\frac{\pi}{6} + 2k\pi, -\frac{5\pi}{6} + 2k\pi \mid k \in Z$

9

الف) $\sqrt{1-\log_{\frac{1}{2}}^{x-1}} \to 1-\log_{\frac{1}{2}}^{x-1} \geq 0 \to -\log_{\frac{1}{2}}^{x-1} \geq -1 \to \log_{\frac{1}{2}}^{x-1} \leq 1$

$\to x-1 > 0 \to x > 1$

$x-1 \leq \frac{1}{2} \to x \leq \frac{3}{2}$ (corrected in blue to $\geq$)

$D = [\frac{3}{2}, +\infty)$

$(1, \frac{3}{2}]$ ← ادامه

ادامه $\to (-\infty, 0] \cup [3, +\infty) - \{-1, 4\}$

(score: 1.5)

ب) $\frac{\sqrt{x^2-x}}{1-\log_4^{x^2-3x}} \to x^2-x \geq 0 \to x(x-1) \geq 0 \to x=0, x=1 \to (-\infty, 0] \cup [1, +\infty)$

$1-\log_4^{x^2-3x} \neq 0$ ، $\log_4^{x^2-3x} \neq 1 \to x^2-3x \neq 4 \to x^2-3x-4 \neq 0$

$\to (x-4)(x+1) \neq 0 \to x \neq 4, -1 \to x^2-3x > 0 \to x(x-3) > 0 \to (-\infty, 0) \cup (3, +\infty)$

10

الف) $\sqrt{2^{4x-x^2}-8} \to 2^{4x-x^2}-8 \geq 0 \to 2^{4x-x^2} \geq 8 = 2^3 \to 4x-x^2 = 3$

$x(4-x) = 3 \to x=3, x=1 \to (-\infty, 1] \cup [3, +\infty)$

(score: 1.75)

ب) $\left(\frac{3x+5}{2x+4}\right)! \to \frac{3x+5}{2x+4} \in W \to 3x+5 = 2wx+4w \to 3x-2wx = 4w-5 \to x = \frac{4w-5}{3-2w} \to \{x \mid x = \frac{4k-5}{3-2k}, k \in$