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(AP)

$$2x^2 + y^2 - \varepsilon x + 6y + k = 0 \Rightarrow \begin{cases} 2x^2 - \varepsilon x = 2(x^2 - \frac{\varepsilon}{2}x) = 2((x - \frac{\varepsilon}{4})^2 - \frac{\varepsilon^2}{16}) = 2(x - \frac{\varepsilon}{4})^2 - \frac{\varepsilon^2}{8} \\ y^2 + 6y = (y^2 + 6y + 9) - 9 = (y + 3)^2 - 9 \end{cases} \quad (1)$$

$$\Rightarrow 2(x - \frac{\varepsilon}{4})^2 - \frac{\varepsilon^2}{8} + (y + 3)^2 - 9 + k = 0 \Rightarrow 2(x - \frac{\varepsilon}{4})^2 + (y + 3)^2 + (k - 11) = 0$$

$$\Rightarrow 2(x - \frac{\varepsilon}{4})^2 + (y + 3)^2 + (k - 11) = 0 \Rightarrow k - 11 = 0 \Rightarrow k = 11$$

در نقطه (1, -3) برابر صفر اند

$$f(x) = \begin{cases} x^2 + \varepsilon x; & x \geq a \\ x^2 + a + x; & x \leq a \end{cases} \rightarrow \begin{cases} a^2 + \varepsilon a = 2a + a + \varepsilon \Rightarrow a^2 + \varepsilon a - 3a - \varepsilon = 0 \\ a^2 + a - \varepsilon = 0 \Rightarrow (a + \varepsilon)(a - 1) = 0 \Rightarrow \begin{cases} a = 1 \checkmark \\ a = -\varepsilon \times \end{cases} \end{cases} \quad (a > 0) \quad (2)$$

$$f(1) = (1)^2 + \varepsilon(1) = 2$$

$$f(1) = (1)^2 + 1 + \varepsilon = 2$$

$$f(x) = \begin{cases} 2x^2 - b; & |x+1| \geq 2 \\ a + x; & -2 \leq x \leq -1 \end{cases} \Rightarrow \begin{cases} x+1 \geq 2 \Rightarrow x \geq 1 \\ x+1 \leq -2 \Rightarrow x \leq -3 \end{cases} \quad (3)$$

a+b=?

$$\Rightarrow 2(-2)^2 - b = a + 2(-2) \Rightarrow 11 - b = a - 4 \Rightarrow a + b = 15$$

$$D = \sqrt{2x-a} + \sqrt{b-x} \Rightarrow \begin{cases} 2x-a \geq 0 \Rightarrow 2x \geq a \Rightarrow x \geq \frac{a}{2} \\ b-x \geq 0 \Rightarrow x \leq b \Rightarrow x \leq \frac{b}{2} \end{cases} \Rightarrow D_f = [\frac{a}{2}, \frac{b}{2}] \Rightarrow \frac{a}{2} = \frac{b}{2} = 2 \quad (4)$$

$$D_f = \{2\}$$

$$\frac{b}{a} = ? \Rightarrow \frac{6}{12} = \frac{1}{2}$$

$$f(x) = \sqrt{\frac{x^2 - x - 6}{2\varepsilon - 14x^2 + 36}} \Rightarrow y = \sqrt{\frac{(x+2)(x-3)}{(2\varepsilon-14)(x-3/2)}} \Rightarrow \begin{matrix} x & -2 & -3/2 & 3/2 & 3 \\ + & \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \end{matrix} \Rightarrow D_f = (-\infty, -2) \cup (-2, 3/2) \cup (3/2, +\infty)$$

$$\Rightarrow y = \sqrt{\frac{x^2 - x^2 - ax + 6}{x^2 + 2x^2 - ax - 6}} \Rightarrow \frac{x^2 - x^2 - ax + 6}{x^2 + 2x^2 - ax - 6} = \frac{-ax + 6}{3x^2 - ax - 6}$$

$$\Rightarrow y = \sqrt{\frac{(x-1)(x-2)(x+1)}{(x+1)(x+2)(x-3)}} \Rightarrow \frac{(x-1)(x-2)(x+1)}{(x+1)(x+2)(x-3)}$$

$$\Rightarrow \frac{x-1}{x+2} \cdot \frac{x-2}{x-3}$$

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$$f(x) = \sqrt{\frac{x+1}{14x-1}} + \sqrt{\frac{6-x}{x^2+2x+6}} \Rightarrow \begin{cases} \frac{x+1}{14x-1} \geq 0 \Rightarrow x \neq 1/14, x+1 \geq 0 \Rightarrow x \geq -1 \\ \frac{6-x}{x^2+2x+6} \geq 0 \Rightarrow x \leq 6 \end{cases} \Rightarrow D_f = [-1, 6] - \{1/14\}$$

$$\Rightarrow \sqrt{[x]-\varepsilon} + \sqrt{2-[x]} \Rightarrow \begin{cases} [x]-\varepsilon \geq 0 \Rightarrow [x] \geq \varepsilon \Rightarrow x \geq \varepsilon \\ 2-[x] \geq 0 \Rightarrow [x] \leq 2 \Rightarrow x < 3 \end{cases} \Rightarrow D_f = \emptyset$$

الف) $y = \sqrt{[-u]-2} \Rightarrow [-u]-2 \geq 0 \Rightarrow [-u] \geq 2 \Rightarrow -u \geq 2 \Rightarrow u \leq -2 \Rightarrow D_f = (-\infty, -2]$

ب) $y = \frac{au^2 + \gamma}{[u]^2 - 2[u] - 2} \Rightarrow [u]^2 - 2[u] - 2 \neq 0 \Rightarrow ([u]-2)([u]+1) \neq 0 \Rightarrow \begin{cases} [u] \neq 2 \Rightarrow [2, \infty) \\ [u] \neq -1 \Rightarrow [-1, 0) \end{cases}$

$\Rightarrow D_f = \mathbb{R} - \{(-1, 0) \cup [2, \infty)\}$

ج) $y = \frac{\cot u + 1}{\tan u + 1} \Rightarrow \begin{cases} \tan u + 1 \neq 0 \Rightarrow \tan u \neq -1 \Rightarrow u \neq k\pi - \frac{\pi}{4} \\ \sin u \neq 0 \Rightarrow u \neq \frac{k\pi}{\pi} \end{cases}$

$\Rightarrow D_f = \mathbb{R} - \left\{ k\pi - \frac{\pi}{4}, \frac{k\pi}{\pi} \right\}$

د) $y = \sqrt{1 - \varepsilon \sin^2 u} \Rightarrow 1 - \varepsilon \sin^2 u \geq 0 \Rightarrow -\varepsilon \sin^2 u \geq -1 \Rightarrow \sin^2 u \leq \frac{1}{\varepsilon} \Rightarrow -\frac{1}{\sqrt{\varepsilon}} \leq \sin u \leq \frac{1}{\sqrt{\varepsilon}}$

$\Rightarrow D_f = \left[k\pi - \frac{\pi}{\sqrt{\varepsilon}}, k\pi + \frac{\pi}{\sqrt{\varepsilon}} \right]$

هـ) $y = \sqrt{1 - \log_{\frac{1}{r}} x + 1} \Rightarrow \begin{cases} x - 1 > 0 \Rightarrow x > 1 \\ 1 - \log_{\frac{1}{r}} (x-1) \geq 0 \Rightarrow \log_{\frac{1}{r}} (x-1) \leq 1 \Rightarrow x-1 \geq \frac{1}{r} \Rightarrow x \geq \frac{r}{r-1} \end{cases}$

$\Rightarrow D_f = \left[\frac{r}{r-1}, +\infty \right)$

و) $y = \frac{\sqrt{x^2 - u}}{1 - \log_{\varepsilon} x^2 - ru} \Rightarrow \begin{cases} x^2 - u \geq 0 \Rightarrow x \geq \sqrt{u} \\ 1 - \log_{\varepsilon} x^2 - ru \neq 0 \Rightarrow \log_{\varepsilon} x^2 - ru \neq 1 \Rightarrow x^2 - ru \neq \varepsilon \Rightarrow x^2 - ru - \varepsilon \neq 0 \\ \Rightarrow (x+1)(x-1) \neq 0 \Rightarrow x \neq -1, x \neq 1 \end{cases}$

$\Rightarrow D_f = ((-\infty, 0] \cup [1, +\infty)) - \{-1, 1\}$

ز) $y = \sqrt{r^{\varepsilon u - u^2} - 1} \Rightarrow r^{\varepsilon u - u^2} - 1 \geq 0 \Rightarrow r^{\varepsilon u - u^2} \geq 1 \Rightarrow r^{\varepsilon u - u^2} \geq r^2$

$\Rightarrow \varepsilon u - u^2 \geq 2 \Rightarrow -u^2 + \varepsilon u - 2 \geq 0 \Rightarrow (-u+1)(u-3) \geq 0 \Rightarrow \begin{cases} u=1 \\ u=3 \end{cases}$

$\Rightarrow D_f = [1, 3]$

ح) $y = \left(\frac{ru + a}{ru + \varepsilon} \right)! \Rightarrow \frac{ru + a}{ru + \varepsilon} \in \mathbb{N} \Rightarrow ru + a = ru + \varepsilon \Rightarrow ru - ru = \varepsilon - a$

$\Rightarrow u = \frac{\varepsilon - a}{r - r} \Rightarrow D_f = \left\{ u \mid u = \frac{rk - a}{r + rk}, k \in \mathbb{N} \right\}$