

$$2x^2 + y^2 - 4x + 4y + k = 0 \rightarrow 2x^2 - 4x + y^2 + 4y + k = 0 \rightarrow 2(x^2 - 2x) + y^2 + 4y + k = 0$$

$$\rightarrow 2(x-1)^2 - 2 + (y+2)^2 - 4 + k = 0 \rightarrow 2(x-1)^2 + (y+2)^2 - 6 + k = 0$$

$$\rightarrow 2(x-1)^2 + (y+2)^2 = 6 - k \rightarrow 6 - k > 0 \rightarrow 6 > k$$

اگر  $6 > k$  باشد ۲ مقدار برای  $x$  و  $y$  بدست می آید غیر قابل قبول است

$$k = 6$$

$$x^2 + 4x = 2x + a + 2 \rightarrow x^2 + 2x = a + 2 \rightarrow x^2 + 2x - a - 2 = 0 \rightarrow (x+2)(x-a) = 0$$

$$\Rightarrow a = -2 \text{ یا } a = 1 \checkmark \rightarrow x^2 + 4x = 2x + 1 + 3 \xrightarrow{x=1} \boxed{f(1) = 5}$$

در  $a = -2$  نیز سلفه

$$|x+1| \geq 2 \rightarrow \begin{cases} x+1 \geq 2 \rightarrow x \geq 1 \\ x+1 \leq -2 \rightarrow x \leq -3 \end{cases}$$

$$\xrightarrow{x=-3} 2(-3)^2 - b = a - 4 \rightarrow 18 - b = a - 4 \rightarrow 22 = a + b$$

$$\sqrt{4x-a} + \sqrt{b-3x} \xrightarrow{x=2} \begin{cases} 4x-a \geq 0 \rightarrow 12-a \geq 0 \rightarrow 12 \geq a \\ b-3x \geq 0 \rightarrow b-6 \geq 0 \rightarrow b \geq 6 \end{cases}$$

$$\Rightarrow \boxed{\frac{b}{a} = \frac{1}{2}} \checkmark$$

$$(الف) \sqrt{\frac{x+1}{|x-3|}} + \sqrt{\frac{4-x}{x^2+2x+2}} \rightarrow \begin{cases} x+1 \geq 0 \rightarrow x \geq -1 \\ 4-x \geq 0 \rightarrow x \leq 4 \end{cases} \rightarrow [-1, 4] - \{3\} = D_f$$

$$(ب) \sqrt{[x]-2} + \sqrt{2-[x]} \rightarrow \begin{cases} [x]-2 \geq 0 \rightarrow [x] \geq 2 \rightarrow x \geq 2 \\ 2-[x] \geq 0 \rightarrow 2 \geq [x] \rightarrow x < 3 \end{cases} \rightarrow 2 \leq x < 3$$

با بدین این دو نامعادله اشتراک بیکی چون دامنه‌ی تابع به ازای هر روی آنها تقریب می‌دهد

$$\text{اشتراک} \rightarrow \emptyset \rightarrow \cap \emptyset = \emptyset$$

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |                    |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------|
| <p>الف) <math>\sqrt{\frac{x^2-x-4}{x^2-13x+34}} \sim \sqrt{\frac{(x-4)(x+1)}{x^2-13x+34}} \sim \sqrt{\frac{(x-4)(x+1)}{(x-9)(x-4)}} \sim \sqrt{\frac{(x-4)(x+1)}{(x+1)(x+2)(x-4)(x+2)}}</math></p> <p><math>\frac{-4}{+} \frac{-1}{-} \frac{1}{+} \frac{1}{+} \frac{2}{+} \frac{2}{+} \rightarrow (-\infty, -4) \cup (1, +\infty) - \{3\}</math></p> <p>ب) <math>\sqrt{x-1} \rightarrow (x-1)(x-4)(x+2)</math></p> <p>ج) <math>\sqrt{x-2} \rightarrow (x-2)(x+1)(x+2)</math></p>                                                                                                                                                                                                                                                                          | <p>2</p> <p>6</p>  |
| <p><math>\sqrt{[-x]-1} \rightarrow [-x]-1 \geq 0 \rightarrow [-x] \geq 1 \rightarrow -x \geq 1 \rightarrow x \leq -1 \rightarrow (-\infty, -1]</math></p> <p>ب) <math>[x]^2 - 2[x] - 3 \neq 0 \rightarrow ([x]-3)([x]+1) \neq 0 \rightarrow [x] \neq 3</math></p> <p><math>\hookrightarrow \mathbb{R} - [3, +\infty)</math></p> <p>ج) <math>[x] \neq -1 \rightarrow \mathbb{R} - [-1, +\infty)</math></p> <p>I, II <math>\sim (-\infty, -1) \cup [0, 3) \cup [4, +\infty)</math></p>                                                                                                                                                                                                                                                                      | <p>2</p> <p>7</p>  |
| <p>الف) <math>\frac{\cos x}{\sin x} + 1 = \frac{\cos x + \sin x}{\sin x} = \frac{\cos x}{\sin x} = \cot x</math></p> <p>ب) <math>\frac{\sin x}{\cos x} + 1 = \frac{\sin x + \cos x}{\cos x} = \frac{\sin x}{\cos x} = \tan x</math></p> <p>ج) <math>\sqrt{1 - \sin^2 x} \rightarrow 1 - \sin^2 x \geq 0 \rightarrow 1 \geq \sin^2 x \rightarrow \frac{1}{\sqrt{2}} \geq \sin x \rightarrow \frac{1}{\sqrt{2}} \geq \sin x</math></p> <p>د) <math>\frac{1}{\sqrt{2}} \geq \sin x \rightarrow x \in [\frac{\pi}{6}, \frac{5\pi}{6}]</math></p> <p>هـ) <math>\frac{1}{\sqrt{2}} \leq -\sin x \rightarrow -\frac{1}{\sqrt{2}} \leq \sin x</math></p> <p>و) <math>\frac{1}{\sqrt{2}} \leq \sin x \rightarrow x \in [\frac{3\pi}{2}, \frac{7\pi}{2}]</math></p> | <p>2</p> <p>8</p>  |
| <p>الف) <math>x-1 &gt; 0 \rightarrow x &gt; 1</math> I</p> <p>ب) <math>\log \frac{x-1}{x} \geq 0 \rightarrow 1 \geq \log \frac{x-1}{x} \rightarrow x-1 \leq \frac{x}{x} \rightarrow x \leq \frac{x}{x} \rightarrow x \leq 1</math> II</p> <p>ج) <math>x^2 - x \geq 0 \rightarrow x(x-1) \geq 0</math></p> <p>د) <math>x^2 - 3x &gt; 0 \rightarrow x(x-3) &gt; 0</math></p> <p>هـ) <math>\log \frac{x^2-3x}{x} \neq 1 \rightarrow x^2-3x \neq x \rightarrow x^2-4x \neq 0 \rightarrow (x-4)(x) \neq 0 \rightarrow x \neq 4, -1</math></p>                                                                                                                                                                                                                  | <p>2</p> <p>9</p>  |
| <p>الف) <math>x^{x-2} - 1 \geq 0 \rightarrow x^{x-2} \geq 1 \rightarrow x^{x-2} \geq 1 \rightarrow x^{x-2} \geq 1 \rightarrow x^{x-2} \geq 1</math></p> <p>ب) <math>(x-1)(x-3) \leq 0</math></p> <p>ج) <math>\frac{x^2+0}{x^2+3} \in \mathbb{W} \rightarrow x = \frac{x^2+0}{x^2+3} \sim  x  \rightarrow x = \frac{x^2+0}{x^2+3}, k \in \mathbb{W}</math></p>                                                                                                                                                                                                                                                                                                                                                                                             | <p>2</p> <p>10</p> |