آروشا یادی یازدهم دفتر A تکلیف شماره ۱۹

۱ـ $\beta , 3\beta \rightarrow$ ریشه‌های معادله $S = \frac{-b}{a} = \frac{a}{3} = \beta + 3\beta = 4\beta \quad P = \frac{c}{a} = \frac{4}{3} = 3\beta(\beta) = 3\beta^2$

$\rightarrow 3\beta^2 = \frac{4}{3} \rightarrow \beta^2 = \frac{4}{9} \rightarrow \beta = \pm \frac{2}{3}$

$\beta = \frac{2}{3} \rightarrow 4\left(\frac{2}{3}\right) = \frac{a}{3} \rightarrow a = 8$

$\beta = -\frac{2}{3} \rightarrow 4\left(\frac{-2}{3}\right) = \frac{a}{3} \rightarrow a = -8 \quad \Longrightarrow |-8-8| = 16$

۲ـ $\sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = \frac{1}{\sqrt{\beta}} + \frac{1}{\sqrt{\alpha}} = \frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}} = \frac{\sqrt{\alpha+\beta+2\sqrt{\alpha\beta}}}{\sqrt{\alpha\beta}} = 5 \quad S = \alpha+\beta = \frac{m+14}{36} \quad P = \alpha\beta = \frac{1}{36}$

$\rightarrow \frac{\sqrt{\frac{m+24}{36}}}{\frac{1}{6}} = 5 \rightarrow \sqrt{\frac{m+24}{36}} = \frac{5}{6} \rightarrow \frac{m+24}{36} = \frac{25}{36} \rightarrow m = -1 \rightarrow -x^2 + 3x + 2 = 0$

$P = \frac{c}{a} = \frac{2}{-1} = -2$

۳ـ $\alpha\beta = -2 , \alpha+\beta = 1 \rightarrow \alpha(1-\alpha) = \alpha - \alpha^2 = -2 \rightarrow \alpha^2 - \alpha - 2 = 0 \rightarrow \alpha = -1 \quad \alpha = 2$

$\alpha = -1 \rightarrow \beta = 2$

$\longrightarrow$ یکی از ریشه‌ها ۲ و دیگری ۱- هست

$\alpha = 2 \rightarrow \beta = -1$

$x = 2 \rightarrow 32 + 4K - 18 - 2 = 0 \rightarrow 4K = -12 \rightarrow \boxed{K = -3}$

$x = -1 \rightarrow -4 + K + 9 - 2 = 0 \rightarrow \boxed{K = -3}$

۴ـ $\alpha+\beta = -6 \quad \alpha\beta = a \quad \alpha = \frac{-6 \pm \sqrt{36-4a}}{2} = -3 \pm \sqrt{9-a} \xrightarrow{\alpha < \beta} \alpha = -3 - \sqrt{9-a} , \beta = -3 + \sqrt{9-a}$

برای منفی بودن ریشه‌ها $\rightarrow -3 + \sqrt{9-a} < 0 \rightarrow \sqrt{9-a} < 3 \rightarrow a > 0 \quad \alpha^2 = 18 - a + 6\sqrt{9-a} \quad \beta^2 = 18 - a - 6\sqrt{9-a}$

$\rightarrow 3\alpha^2 + 2\beta^2 = 3(18 - a + 6\sqrt{9-a}) + 2(18 - a - 6\sqrt{9-a}) = 90 - 5a + 6\sqrt{9-a} = 12\sqrt{2} + 85$

$\rightarrow 6\sqrt{9-a} = 12\sqrt{2} + 5a - 5 \xrightarrow{\text{به توان ۲}} 120\sqrt{2}(a-1) = 0 \rightarrow a - 1 = 0 \rightarrow \boxed{a = 1}$

$\alpha = -3 - \sqrt{8} < \beta = -3 + \sqrt{8} < 0 \quad \checkmark$

۵ـ $x^2 = n \rightarrow n^2 - 7n - 5 = 0 \rightarrow \Delta = 49 + 20 = 69 \quad n = \frac{7 \pm \sqrt{69}}{2} \rightarrow x^2 = \frac{7 - \sqrt{69}}{2}$ غ ق ق

$x^2 = \frac{7 + \sqrt{69}}{2} \rightarrow x = \pm\sqrt{\frac{7+\sqrt{69}}{2}} \rightarrow S = 0 \quad P = \frac{7+\sqrt{69}}{2} \rightarrow P^2 = \frac{49 + 14\sqrt{69} + 69}{4}$

$2P^2 - 3SP + 2S = 59 + 7\sqrt{69}$

r_1 , r_2 : ریشه‌های معادله دوم $r_1+\frac{1}{2} , r_2+\frac{1}{2}$: ریشه‌های معادله اول ۶

$S_2 = r_1+r_2 = \frac{-1}{2}$ $P_2 = r_1 \cdot r_2 = \frac{-3}{a}$ — $S_1 = r_1+\frac{1}{2}+r_2+\frac{1}{2} = (r_1+r_2)+1 = \frac{-1}{2}+1 = \frac{1}{2} = \frac{a}{2} \to \boxed{a=1}$

$P_1 = (r_1+\frac{1}{2})(r_2+\frac{1}{2}) = r_1 \cdot r_2 + \frac{1}{2}(r_1+r_2)+\frac{1}{4} = \frac{-3}{a} - \frac{1}{4}+\frac{1}{4} = \frac{-3}{a} \xrightarrow{a=1} P_1 = -3 = \frac{b}{2} \to \boxed{b=-6}$

$\left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = \left[\frac{-3}{2}\right] = -2$

$S = \frac{a}{a} = 1 = \alpha+\beta$ $P = \frac{-b}{a} = \alpha \cdot \beta \to \alpha = 1-\beta \to \alpha^2 = (1-\beta)^2 = 1-2\beta+\beta^2$ ۷

$40\beta^2 + 20(1-2\beta+\beta^2) - 20\beta = 17 \to 60\beta^2 - 60\beta + 3 = 0 \xrightarrow{\div 3} 20\beta^2 - 20\beta + 1 = 0$

$\Delta = 400 - 80 = 320 \to \beta = \frac{20 \pm 8\sqrt{5}}{40} = \frac{5 \pm 2\sqrt{5}}{10} , \alpha = \frac{5 \mp 2\sqrt{5}}{10}$

$|\alpha-\beta| = \left|\frac{(5-2\sqrt{5})-(5+2\sqrt{5})}{10}\right| = \frac{2\sqrt{5}}{5}$

$x^2-(a+1)x+a=0 \to \alpha = 2n-1 , \beta = 2n+1 , n \in N$ $\alpha+\beta = S = 4n = a+1 \to a = 4n-1$ ۸

$P = \alpha \cdot \beta = (2n-1)(2n+1) = 4n^2-1 = a \to 4n^2 = 4n \to n^2 = n \to n = 1 \to a = 3 \to P_1 = 3$

$x^2-(3a+1)x+b=0 \xrightarrow{a=3} x^2-10x+b=0 \to 2K , 2K+2 \to S_2 = 2K+2K+2 = 4K+2 = 10 \to K = 2$

ریشه‌های معادله اول $= 1 , 3$ ریشه‌های معادله دوم $= 4 , 6$ $|(1\times3)-(4\times6)| = |3-24| = 21$

$S = \alpha+\beta = \frac{-1}{1} = -1$ $P = \alpha \cdot \beta = \frac{-1-m^2}{1} = -1-m^2$ $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$ ۹

$\to 1-2(-1-m^2) = 1+2+2m^2 = 3+2m^2$ $m^2 \geq 0 \to 2m^2 \geq 0 \to 2m^2+3 \geq 3 \to \alpha^2+\beta^2 \geq 3$

$\Rightarrow m^2 = 0 \to 2(0)+3 = 3 \Rightarrow \alpha^2+\beta^2 = 3$ کمترین مقدار وقتی حاصل می‌شود که $m^2=0$ باشد

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۱۰. چون دو نقطه هم‌عرض هستند طول رأس سهمی برابر میانگین طول دو نقطه است. $\frac{-5+3}{2} = \frac{-2}{2} = -1 \to S(-1,1)$

$\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 5$ $x_S = \frac{-b}{2a} = -1 \to \frac{-b}{a} = -2 = S = \alpha+\beta$

$\to (-2)^2 - 2\alpha\beta = 5 \to 2\alpha\beta = 4-5 = -1 \to \alpha\beta = \frac{-1}{2} = P$ $y = x^2 - Sx + P \to y = x^2+2x-\frac{1}{2}$

$\xrightarrow{x=0} y = 0+0-\frac{1}{2} \to y = \frac{-1}{2} \to (0, \frac{-1}{2})$