Date: Sub: ۱۷ تکلیف: ۱۹ * ترگل آیین

①

$3x^2 - ax + 4 = 0$ $S = \alpha + 3\alpha = 4\alpha = \frac{a}{3}$

$P = \alpha \times 3\alpha = 3\alpha^2 = \frac{4}{3} \to \alpha^2 = \frac{4}{9} \to \alpha = \pm\frac{2}{3}$ (2)

$4 \times \frac{2}{3} = \frac{a}{3} \to a = 8 \longrightarrow 8 - (-8) = 16$

$4 \times \frac{-2}{3} = \frac{a}{3} \to a = -8$

②

$36x^2 - (m+14)x + 1 = 0$ $mx^2 + 3x + 2 = 0$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\alpha} + \sqrt{\beta}}{\underset{\to \frac{1}{6}}{\sqrt{\alpha\beta}}} = 5 \to \sqrt{\alpha} + \sqrt{\beta} = \frac{5}{6} \longrightarrow$

$\alpha + \beta + 2\sqrt{\alpha\beta} = \frac{25}{36} \longrightarrow$

$\frac{m+14}{36} + \frac{2}{6} = \frac{25}{36} \longrightarrow m + 14 + 12 = 25 \to m = -1$ (2)

$\frac{2}{m} = \frac{2}{-1} = -2$

③

$4x^3 + Kx^2 - 9x - 2 = 0$

$\alpha + \beta = 1 \to \alpha + \frac{-2}{\alpha} = 1 \to \alpha^2 - \alpha - 2 = 0$

$\alpha\beta = -2 \to \beta = \frac{-2}{\alpha} \longrightarrow \alpha = 2 \longrightarrow$ (2)

$\beta = -1 \longrightarrow x = 2$

$32 + 4K - 18 - 2 = 0 \to K = -3$

④

$\alpha + \beta = -6$, $a\beta = a$

$a < \beta < 0 \to 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \to \frac{5}{2}(\alpha^2 + \beta^2) + \frac{1}{2}(\alpha^2 - \beta^2)$

$\to \alpha^2 + \beta^2 = 36 - 2\alpha\beta$, $\underset{SD}{\alpha^2} - \beta^2 = -6\sqrt{\Delta}$

$12\sqrt{2} + 85 = \frac{5}{2}(36 - 2\alpha\beta) + \frac{1}{2}(-6\sqrt{\Delta}) \to$ (2)

$24\sqrt{2} = -6\sqrt{\Delta} \to \sqrt{\Delta} = -4\sqrt{2}$, $34 - 36 = -2\alpha\beta \to$

$\alpha\beta = 1 \to P = 1 = \frac{a}{1}$

Elipon $\to a = 1$

⑤

$x^4 - 7x^2 - 5 = 0 \longrightarrow t = x^2 \longrightarrow t^2 - 7t + 5 = 0$

$\xrightarrow{t>0} \dfrac{7+\sqrt{49+20}}{2} \longrightarrow t = \dfrac{7+\sqrt{69}}{2} = x^2 \longrightarrow x = \pm\sqrt{\dfrac{7+\sqrt{69}}{2}}$

$\rightarrow S = 0 \; , \; P = \dfrac{-7-\sqrt{69}}{2} \longrightarrow 2P^2 - 3SP + 2S = 2P^2 =$ (5)

$2x^2 - ax + b = 0 \begin{cases} S = \frac{a}{2} \\ P = \frac{b}{2} \end{cases}$ $\quad 2ax^2 + ax - 6 \begin{cases} S = -\frac{1}{2} \to \alpha+\beta = \frac{1}{2} = \frac{a}{2} \to a = 1 \\ P = -3 \to b = -6 \end{cases}$

$\dfrac{118+14\sqrt{69}}{2} = 59 + 7\sqrt{69}$

$\left[\frac{ab}{4}\right] = -2$

⑥

$2x^2 - ax + b = 0 \qquad \alpha, \beta \rightarrow \quad S = \dfrac{a}{2} \qquad P = \dfrac{b}{2}$

$2ax^2 + ax - 6 = 0 \rightarrow \alpha + \frac{1}{2}, \; \beta + \frac{1}{2} \rightarrow S' = \dfrac{-a}{2a} = \dfrac{-1}{2}$ (0.25)

$P' = \dfrac{-6}{2a} = -\dfrac{3}{a} \; , \; S' = \alpha + \beta + 1 \rightarrow -\frac{1}{2} = \frac{a}{2} + 1 \rightarrow a = -3$

$P' = (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \frac{b}{2} + \frac{1}{4} - \frac{3}{4} = -\dfrac{3}{a} = 1 \rightarrow b = 3 \rightarrow$ $(a \rightarrow -3)$

$\left[\frac{ab}{4}\right] = \left[\frac{-9}{4}\right] = -3$

⑦

$ax^2 - ax - b = 0 \qquad 40\beta^2 + 20\alpha^2 - 20\alpha = 17$

$\longrightarrow x^2 = \dfrac{ax+b}{a} \rightarrow \beta^2 = \dfrac{a\beta + b}{a} \rightarrow$ (0.5)

$\alpha = 1 - \beta \quad 40\beta^2 + 20(1-\beta)^2 - 20\beta = 7 \Rightarrow 20\beta\underbrace{(\beta - 1)}_{-\alpha} = -3 \rightarrow \alpha\beta = \frac{1}{20} = \frac{-b}{a}$

$\rightarrow a = -20b \quad ax^2 - ax - b = 0 \rightarrow -20bx^2 + 20bx - b = 0 \quad |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{2}{\sqrt{20}}$

$40\dfrac{a\beta + b}{a} + 20\dfrac{(a\alpha + b)}{a} - 20\beta = 17 \rightarrow 20a + 40b = 17a$

$\rightarrow b = \dfrac{-3}{40}a \longrightarrow D = \dfrac{\sqrt{\Delta}}{|a|} = \dfrac{\sqrt{a^2 + 4ax - \frac{3a}{40}}}{|a|} \rightarrow$

$= \sqrt{\dfrac{7}{10}}$

⑧

$S = a + 1 = 2\alpha + 2 \longrightarrow \alpha^2 + 2\alpha + 1 = 2\alpha + 2 \rightarrow$

$P = a = \alpha^2 + 2\alpha \qquad \alpha = \pm 1 \xrightarrow{\alpha \in \mathbb{N}} \alpha = 1, a = 3$

$S = 10 = 2\beta + 2 \rightarrow \beta = 4 \; , b = 6 \times 4 = 24$ (5)

$b - a = 24 - 3 = 21$

(9)

$\alpha+\beta = \frac{-b}{a} = -1$

$\alpha\beta = -1 - m^2$

$\rightarrow \alpha^2+\beta^2 = 1 - 2(-1-m^2) = 2m^2+3 \xrightarrow{m^2 \geq 0} \underline{\min = 3}$

(10)

$A, B \rightarrow$ رأس سهمی $\rightarrow x_{\text{رأس}} = \frac{-a+3}{2} = -1 \rightarrow y_{\text{رأس}} = 1$

$\rightarrow y-1 = a(x+1)^2 \rightarrow -\frac{1}{a} = (x+1)^2$

$t = \sqrt{\frac{-1}{a}} \qquad x = -1 \pm t \qquad \alpha = -1+t$

$\beta = -1-t$

$\alpha^2+\beta^2 = 5 \rightarrow (-1+t)^2 + (-1-t)^2 = 2+2t^2 \rightarrow$

$2t^2+2 = 5 \rightarrow t^2 = \frac{3}{2} \rightarrow \frac{-1}{a} = \frac{3}{2} \rightarrow a = \frac{-2}{3}$

$\xrightarrow{x=0} y - 1 = a \rightarrow \underline{y = \frac{1}{3}} \rightarrow$ عرض از مبدأ