$3x^2 - ax + 4 = 0$ $\qquad \alpha = 3\beta \qquad |a_2 - a_1| = ?$ $\qquad a = 8,\ a_2 = -\frac{12}{3}$ -1

$\hookrightarrow S = 4\alpha \quad , P = 3\alpha^2$ $\qquad a = 12\alpha$

روش اول $\to x^2 - Sx + P \to x^2 - 4\alpha x + 3\alpha^2 = 0$

$3x^2 - ax + 4 = 0$ $\to -a = -12\alpha$, $9\alpha^2 = 4 \hookrightarrow \alpha = \pm\frac{2}{3}$

$|a_2 - a_1| = |-8 - 8| = |-16| = 16$

-2

$36x^2 - (m+14)x + 1 = 0 \qquad \sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5$

$mx^2 + 3x + 2 = 0 \to x_1 x_2 = ?$

$\hookrightarrow P = \frac{1}{36}, \quad S = \frac{m+14}{36}$

$\hookrightarrow \frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\hookrightarrow \frac{\beta + \alpha}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\hookrightarrow \frac{S}{P} + 2\sqrt{\frac{1}{P}} = \frac{\frac{m+14}{36}}{\frac{1}{36}} + 2\sqrt{\frac{1}{\frac{1}{36}}}$

$\hookrightarrow m = -1$ $\qquad$ اهمیت ندارد

$\to -x^2 + 3x + 2 = 0$

$\hookrightarrow P = \frac{c}{a} = \frac{2}{-1} = -2$

-3

$4x^3 + kx^2 - 9x - 2 = 0 \qquad \alpha + \beta = 1 \qquad \alpha\beta = -2$

$\hookrightarrow \underbrace{4(x-\alpha)(x-\beta)}_{x^2 - (\alpha+\beta)x + \alpha\beta}(x - z) \to 4(x^2 - (\alpha+\beta)x + \alpha\beta)(x - z) \to 4(x^3 - (\alpha+\beta+z)x^2 + (\alpha\beta + z(\alpha+\beta))x - \alpha\beta z) = 0$

ریشه سوم

$(4x^3 \underbrace{- 4(\alpha+\beta+z)}_{k} x^2 + \underbrace{4(\alpha\beta + z(\alpha+\beta))}_{-9} x \underbrace{- 4\alpha\beta z}_{-2}) = 0$

$\alpha\beta = -2,\ \alpha + \beta = 1 \to \begin{cases} -4(1+z) = k \\ 4(-2+z) = -9 \\ -4(-2)z = -2 \end{cases} \to z = -\frac{1}{4},\ k = -3$

-4

$x^2 + 6x + a = 0 \qquad \alpha < \beta < 0 \qquad 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85$

$a = ? \to 1$

$\hookrightarrow 3(-6-\beta)^2 + 2(\beta)^2 = 12\sqrt{2} + 85$

$S = -6 \to \alpha = -6 - \beta$

$\hookrightarrow 108 + 36\beta + 3\beta^2 + 2\beta^2 = 12\sqrt{2} + 85$

$\hookrightarrow \alpha = -3 - z\sqrt{2} \qquad \beta = -3 + z\sqrt{2}$

$\left.\begin{array}{l} \alpha^2 = 9 + 6z\sqrt{2} + 2z^2 \\ \beta^2 = 9 - 6z\sqrt{2} + 2z^2 \end{array}\right\} \to 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \to z = 2$

$P = \frac{c}{a} = a = \alpha\beta \to a = (-3 + 2\sqrt{2})(-3 - 2\sqrt{2}) = 9 - 8 = 1$

۵-

$x^4 - 7x^2 - 5 = 0$ $\qquad 2P^2 - 3SP + 2S = ?$

$x^2 = t \;\}\; t^2 - 7t - 5 = 0 \to x^2 = t \to x = \sqrt{t} \to x = \pm\sqrt{\frac{7+\sqrt{69}}{2}}$

$\to t = \frac{7 \pm \sqrt{69}}{2}$

$P = \sqrt{\frac{7+\sqrt{69}}{2}} \times -\sqrt{\frac{7+\sqrt{69}}{2}} \to -\sqrt{\frac{7+\sqrt{69}}{4}}$ $\qquad \frac{59+7\sqrt{69}}{\cancel{}}$

$S = 0$

$3SP, 2S = 0 \to 2P^2 = 2\left(-\sqrt{\frac{7+\sqrt{69}}{4}}\right)^2$

اهمیت ندارد

۶-

$2x^2 - ax + b = 0$ $\qquad \left[\frac{ab}{4}\right] = ?$

$2ax^2 + ax - 6 = 0$

$\to x_1 + 0.5 ,\; x_2 + 0.5 \to S = x_1 + x_2 + 1 \to S = \frac{1}{2}$

$\to x_1 , x_2 \to S = x_1 + x_2 = -\frac{1}{2}$ $\qquad \Big\}\; a = 1$

$x_1 \times x_2 \to -3$

$(x_1 + 0.5)(x_2 + 0.5) = \cancel{x_1 x_2} + \cancel{0.5x_1 + 0.5x_2} + 0.25 = -3$

$\frac{b}{2} = -3 \to b = -6 \to \left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = [-1.5] = -2$

۷-

$ax^2 - ax - b = 0$ $\qquad \beta - \alpha = ?$

$40\beta^2 + 20\alpha^2 - 20\beta = 17$ $\to S = \frac{a}{a} = 1 \quad P = \frac{-b}{a}$

$2\beta^2 + \alpha^2 - \beta = \frac{17}{20}$

$\to 2\beta^2 + 1 + \beta^2 - 2\beta - \beta = \frac{17}{20}$

$\to 3\beta^2 - 3\beta + 1 = \frac{17}{20} \to 3\beta^2 - 3\beta + \frac{3}{20} = 0$

$\to 60\beta^2 - 60\beta + 3 = 0$

ریشه های این معادله همان ریشه های معادله ی اصلی اند (تبدیل)

پس $P = P' \to P' = \frac{1}{20}$ $\Big\}$ $(\alpha - \beta)^2 = (\alpha + \beta)^2 - \frac{4}{20}$

$\to 1 - \frac{4}{20} = \frac{4}{5} \to (\alpha - \beta)^2 = \frac{4}{5}$

$\to \alpha - \beta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

۸-

$x^2-(a+1)x+a=0$ $\quad \alpha, \alpha+2 \to$ ریشه

$x^2-(3a+1)x+b=0$ $\quad \beta, \beta+2 \to$ ریشه

با $\to$ $\alpha+\alpha+2 = 2\alpha+2 = a+1 \to a=2\alpha+1$ $\quad$ $\alpha(\alpha+2)=a$ $\Big\}$ $a=3 \to P_1=3$

$\alpha^2+2\alpha = 2\alpha+1 \to \alpha^2-2\alpha+1=0 \quad (\alpha-1)^2 \to \alpha=1$

$\beta+\beta+2 = 2\beta+2 = 3\overset{3}{\cancel{\alpha}}+1 \to 2\beta+2=10 \to \beta=4 \quad P_2=24$

$P_2-P_1=24-3=21$

[illegible]

۹-

$x^2+x-1-m^2=0$ $\quad \min(\alpha^2+\beta^2)=?$

ریشه‌ها $x = \frac{-1+\sqrt{1+4(1+m^2)}}{2} = \frac{-1\pm\sqrt{5+4m^2}}{2}$

$\alpha = \frac{-1+\sqrt{5+4m^2}}{2} \quad \beta = \frac{-1-\sqrt{5+4m^2}}{2}$

$\alpha^2+\beta^2 = \frac{(-1)^2+(5+4m^2)+(-1)^2+(5+4m^2)}{4} = 3+2m^2$

چون $m\geq 0 \to (h^2+k^2)_{min} = 3$

۱۰-

$A(4,y) \quad B(-5,y) \quad y_{\text{رأس}}=1 \quad x_0=\alpha, \beta$

$\alpha^2+\beta^2=5 \quad y_0=?$

$x_s=-1 \quad y=a(x+1)^2+1$

$y_s=1 \quad y=ax^2+2ax+a+1$

$\alpha+\beta=\frac{-2a}{a} \quad \underset{5}{\cancel{\alpha^2+\beta^2}}+2\alpha\beta=4 \to 2\alpha\beta=-1 \to \alpha\beta=-\frac{1}{2}$

$\frac{a+1}{a}=-\frac{1}{2} \to a=-\frac{2}{3}$ عرض از مبدا $\to a+1=-\frac{2}{3}+\frac{3}{3}=\frac{1}{3}$