۲۰

-۱

$3x^2 - ax + 4 = 0 \qquad \alpha = 3\beta \qquad |a_2 - a_1| = ?$

$\hookrightarrow S = 4\alpha \ , \ P = 3\alpha^2$

$a_1 = 8 \ , \ a_2 = -\frac{16}{2}$

روش دوم $\longrightarrow x^2 - Sx + P \longrightarrow x^2 - 4\alpha x + 3\alpha^2 = 0$

$3x^2 - ax + 4 = 0 \longrightarrow -a = -12\alpha \longrightarrow a = 12\alpha$

$9\alpha^2 = 4 \hookrightarrow \alpha = \pm\frac{2}{3}$

$|a_2 - a_1| = |-8-8| = |-16| = 16$

-۲

$36x^2 - (m+14)x + 1 = 0 \qquad \sqrt{\frac{1}{\alpha}} + \sqrt{\frac{1}{\beta}} = 5$

$mx^2 + 3x + 2 = 0 \longrightarrow x_1 x_2 = ?$

$P = \frac{1}{36} \ , \ S = \frac{m+14}{36}$

$\hookrightarrow \frac{1}{\alpha} + \frac{1}{\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\hookrightarrow \frac{\beta+\alpha}{\alpha\beta} + 2\sqrt{\frac{1}{\alpha\beta}} = 25$

$\hookrightarrow \frac{S}{P} + 2\sqrt{\frac{1}{P}} = \frac{\frac{m+14}{36}}{\frac{1}{36}} + 2\sqrt{\frac{1}{\frac{1}{36}}}$

$\hookrightarrow m = -1$

$-x^2 + 3x + 2 = 0$

$\hookrightarrow P = \frac{c}{a} = \frac{2}{-1} = -2$

[illegible]

-۳

$4x^3 + kx^2 - 9x - 2 = 0 \qquad \alpha + \beta = 1 \qquad \alpha\beta = -2$

ریشه سوم

$\hookrightarrow 4\underbrace{(x-\alpha)(x-\beta)}_{x^2-(\alpha+\beta)x+\alpha\beta}(x-z) \rightarrow 4(x^2-(\alpha+\beta)x+\alpha\beta)(x-z) \rightarrow 4(x^3-(\alpha+\beta+z)x^2 + (\alpha\beta+z(\alpha+\beta))x - \alpha\beta z) = 0$

$(4x^3 \underbrace{-4(\alpha+\beta+z)}_{k}x^2 + \underbrace{4(\alpha\beta+z(\alpha+\beta))}_{-9}x \underbrace{-4\alpha\beta z}_{-2}) = 0$

$\alpha\beta = -2 \ , \ \alpha+\beta = 1 \longrightarrow \left.\begin{array}{l} -4(1+z) = k \\ 4(-2+z) = -9 \\ -4(-2)z = -2 \end{array}\right\} \longrightarrow z = -\frac{1}{4} \ , \ k = -3$

-۴

$x^2 + 6x + a = 0 \qquad \alpha < \beta < 0 \qquad 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85$

$a = ? \rightarrow 1$

$S = -6 \rightarrow \alpha = -6 - \beta$

$\hookrightarrow 3(-6-\beta)^2 + 2(\beta)^2 = 12\sqrt{2} + 85$

$\hookrightarrow 108 + 36\beta + 3\beta^2 + 2\beta^2 = 12\sqrt{2} + 85$

$\hookrightarrow \alpha = -3 - z\sqrt{2} \qquad \beta = -3 + z\sqrt{2}$

$\left.\begin{array}{l} \alpha^2 = 9 + 6z\sqrt{2} + 2z^2 \\ \beta^2 = 9 - 6z\sqrt{2} + 2z^2 \end{array}\right\} \longrightarrow 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85 \longrightarrow z = 2$

$P = \frac{c}{a} = a = \alpha\beta \rightarrow a = (-3+2\sqrt{2})(-3-2\sqrt{2}) = 9 - 8 = 1$

۵-

$x^4 - 7x^2 - 5 = 0 \qquad 3p^2 - 4sp + 2s = ?$

$x^2 = t \;\}\; t^2 - 7t - 5 = 0 \to x^2 = t \to x = \sqrt{t} \to x = \sqrt{\frac{7+\sqrt{69}}{2}}$

$\hookrightarrow t = \frac{7 \pm \sqrt{69}}{2}$

$P = \sqrt{\frac{7+\sqrt{69}}{2}} \times -\sqrt{\frac{7+\sqrt{69}}{2}} \to -\sqrt{\frac{7+\sqrt{69}}{4}} \qquad \frac{59+7\sqrt{69}}{\cancel{\quad}}$ ۲

$S = 0$

$4sp, 2s = 0 \to 3p^2 = 3\left(-\sqrt{\frac{7+\sqrt{69}}{4}}\right)^2$

اشتباه است

۶-

$2x^2 - ax + b = 0 \qquad \left[\frac{ab}{4}\right] = ?$

$2ax^2 + ax - 6 = 0$

$\to x_1 + 0.5 \quad x_2 + 0.5 \to S = x_1 + x_2 + 1 \to S = \frac{1}{2}$

$\to x_1, x_2 \to S = x_1 + x_2 = -\frac{1}{2}$ $\Big\}\; a = 1$ ۲

$x_1 \times x_2 \to -3$

$(x_1+0.5)(x_2+0.5) = \underbrace{x_1x_2}_{-3} + \underbrace{0.5x_1}_{-\frac{1}{4}} + \underbrace{0.5x_2}_{\frac{1}{4}} + 0.25 = -3$

$\frac{b}{2} = -3 \to b = -6 \to \left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = [-1.5] = -2$

۷-

$ax^2 - ax - b = 0 \qquad \beta - \alpha = ?$

$\underline{40\beta^2 + 20\alpha^2 - 20\beta = 17} \qquad \to S = \frac{a}{a} = 1 \quad P = \frac{-b}{a}$ ۲

$2\beta^2 + \alpha^2 - \beta = \frac{17}{20}$

$\hookrightarrow 2\beta^2 + 1 + \beta^2 - 2\beta - \beta = \frac{17}{20} \to 3\beta^2 - 3\beta + 1 = \frac{17}{20} \to 3\beta^2 - 3\beta + \frac{3}{20} = 0$

$\hookrightarrow 60\beta^2 - 60\beta + 3 = 0$

↳ ریشه های این معادله همان ریشه های معادله ی اصلی اند (تبدیل)

پس $P = P' \to P' = \frac{1}{20}$ $\Big\}$ $(\alpha-\beta)^2 = (\alpha+\beta)^2 - \frac{4}{20}$

$\hookrightarrow 1 - \frac{4}{20} = \frac{4}{5} \to (\alpha-\beta)^2 = \frac{4}{5}$

$\hookrightarrow \alpha - \beta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

۸-

$x^2-(a+1)x+a=0$ $\alpha, \alpha+2 \to$ ریشه

$x^2-(3a+1)x+b=0$ $\beta, \beta+2 \to$ ریشه

د)

جمع $\to$ $\alpha+\alpha+2=2\alpha+2=a+1 \to a=2\alpha+1$ $\}$ $a=3 \to P_1=3$

$\alpha(\alpha+2)=a$

$\alpha^2+2=2\alpha+1 \to \alpha^2-2\alpha+1=0\ (\alpha-1)^2 \to \alpha=1$

$\beta+\beta+2=2\beta+2=3\overset{3}{\cancel{\alpha}}+1 \to 2\beta+2=10 \to \beta=4$ $P_2=24$

$P_2-P_1=24-3=21$

احمد رضایی

۹-

$x^2+x-1-m^2=0$ $\min(\alpha^2+\beta^2)=?$

ر $x=\dfrac{-1\pm\sqrt{1+4(1+m^2)}}{2}=\dfrac{-1\pm\sqrt{5+4m^2}}{2}$

د)

$\to$ $\alpha=\dfrac{-1+\sqrt{5+4m^2}}{2}$ $\beta=\dfrac{-1-\sqrt{5+4m^2}}{2}$

$\alpha^2+\beta^2=\dfrac{(-1)^2+(5+4m^2)+(-1)^2+(5+4m^2)}{4}=3+2m^2$

چون $m\geq 0 \to (h^2+k^2)_{min}=3$

۱۰-

$A(4,y)$ $B(-5,y)$ $y_{\text{رأس}}=1$ $x_0=\alpha,\beta$

$\alpha^2+\beta^2=5$ $y_0=?$

د)

$x_s=-1$ $y=a(x+1)^2+1$

$y_s=1$ $y=ax^2+2ax+a+1$

$\alpha+\beta=\dfrac{-2a}{a}$ $\dfrac{\cancel{\alpha^2+\beta^2}}{5}+2\alpha\beta=4 \to 2\alpha\beta=-1 \to \alpha\beta=\dfrac{-1}{2}$

$\dfrac{a+1}{a}=\dfrac{-1}{2} \to a=-\dfrac{2}{3}$ عرض از مبدأ $\to a+1=-\dfrac{2}{3}+\dfrac{3}{3}=\dfrac{1}{3}$