

$$3x^2 - ax + 4 = 0$$

$$\alpha = 3\beta, \alpha + \beta = \frac{a}{3}, \alpha\beta = \frac{4}{3}$$

$$3\beta + \beta = \frac{a}{3} \rightarrow 4\beta = \frac{a}{3} \rightarrow a = 12\beta \rightarrow a = \pm 12 \rightarrow 12(\pm \frac{1}{3}) = \pm 4 \rightarrow 4 - (-4) = 8$$

$$3\beta(\beta) = \frac{4}{3} \rightarrow 3\beta^2 = \frac{4}{3} \rightarrow \beta^2 = \frac{4}{9} \rightarrow \beta = \pm \frac{2}{3}$$

$$3x^2 - (m+1)x + 1 = 0$$

$$\frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}} = \frac{a}{3} \Rightarrow \frac{\sqrt{r_1} + \sqrt{r_2}}{\sqrt{r_1 r_2}} = \frac{\sqrt{r_1 + r_2 + 2\sqrt{r_1 r_2}}}{\sqrt{r_1 r_2}} = \frac{\sqrt{S + \frac{1}{3}}}{\frac{1}{3}} = \frac{a}{3} \Rightarrow \sqrt{S + \frac{1}{3}} = \frac{a}{9}$$

$$(\sqrt{r_1} + \sqrt{r_2})^2 = r_1 + r_2 + 2\sqrt{r_1 r_2} \rightarrow P = \frac{1}{36}$$

$$\rightarrow S = \frac{m+1}{36} = \frac{13}{36} \rightarrow m = -1$$

$$mx^2 + 3x + 2 = 0 \rightarrow P = \frac{2}{m} = -2 \rightarrow m = -1 \Rightarrow S + \frac{1}{3} = \frac{13}{36} \Rightarrow S = \frac{13}{36}$$

$$fx^2 + 4x^2 - 9x - 2 = 0 \rightarrow x^2 - (\alpha + \beta)x + \alpha\beta = x^2 - (1)x + (-2) = x^2 - x - 2$$

$$\alpha + \beta = 1$$

$$\alpha\beta = -2$$

$$\rightarrow (x^2 - x - 2)(px + q) \rightarrow px^2 - px + qx^2 - qx - 2px - 2q$$

$$\rightarrow px^2 + (q-p)x + (-q-2p)x - 2q \rightarrow P = 1, q-p = -1 \rightarrow q = 0, p = 1$$

$$-q-2p = -1 \rightarrow -0-2(1) = -2 \neq -1 \rightarrow q = 1, p = 0$$

$$\Rightarrow k = q - p \rightarrow 1 - 0 = 1$$

$$x^2 + 9x + a = 0$$

$$a < b < 0$$

$$A = \alpha\beta = \alpha(-9-\alpha) = -\alpha^2 - 9\alpha \rightarrow -(\alpha^2 + 9\alpha + \frac{81}{4}) + \frac{81}{4} = -(\alpha + \frac{9}{2})^2 + \frac{81}{4}$$

$$\rightarrow A = 1$$

$$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 18 \rightarrow 3\alpha^2 + 2(-9-\alpha)^2 = 12\sqrt{2} + 18 \Rightarrow 3\alpha^2 + 2(81 + 18\alpha + \alpha^2) = 12\sqrt{2} + 18$$

$$\alpha + \beta = -9, \alpha\beta = A$$

$$\alpha^2 + 18\alpha + 81$$

$$\beta = -9 - \alpha$$

$$\Rightarrow 3\alpha^2 + 2(81 + 18\alpha + \alpha^2) = 12\sqrt{2} + 18 \Rightarrow 5\alpha^2 + 36\alpha + 162 = 12\sqrt{2} + 18$$

$$\rightarrow \alpha = \frac{-36 \pm \sqrt{36^2 - 4 \cdot 5 \cdot (162 - 12\sqrt{2} + 18)}}{10} = \frac{-36 \pm \sqrt{1440 - 360\sqrt{2}}}{10}, \beta = \frac{-27 \pm \sqrt{1440 - 360\sqrt{2}}}{10}$$

$$x^2 - 4x^2 - 5 = 0 \xrightarrow{\frac{0}{2}}, +\sqrt{\frac{V+\sqrt{94}}{2}}, -\sqrt{\frac{V+\sqrt{94}}{2}} \Rightarrow \sqrt{\frac{V+\sqrt{94}}{2}} - \sqrt{\frac{V+\sqrt{94}}{2}} = 0$$

$$2p^2 - 3sp + 2s = ?$$

$$\Rightarrow \left(\sqrt{\frac{V+\sqrt{94}}{2}} \right) \left(\sqrt{\frac{V+\sqrt{94}}{2}} \right) = -\frac{V+\sqrt{94}}{2} \Rightarrow \frac{-V-\sqrt{94}}{2}$$

$$\rightarrow S = 0, p = \frac{-V-\sqrt{94}}{2} \Rightarrow 2\left(\frac{-V-\sqrt{94}}{2}\right)^2 - 3(0)\left(\frac{-V-\sqrt{94}}{2}\right) + 2(0)$$

$$= \frac{V^2 + 2V\sqrt{94} + 94}{2}$$

$$rx^r - ax + b = 0$$

$$ra x^r + ax - r = 0$$

$$\left[\frac{ab}{r} \right] = \left[\frac{-r}{r} \right] = -1$$

$$\alpha + \beta = \alpha' + \beta' + 1 \rightarrow \frac{\alpha}{r} = \frac{-a}{ra} + 1 = \frac{1}{r} \rightarrow a = 1 \rightarrow rx^r + x - r = 0 \rightarrow a', \beta' = -r, \frac{r}{r} \rightarrow \alpha, \beta = -\frac{r}{r}, r$$

$$\rightarrow \frac{b}{r} = \alpha\beta = -r \rightarrow b = -r \rightarrow \left[\frac{(1)(-r)}{r} \right] = -1$$

$$A x^r - a x - b = 0$$

$$\rightarrow \alpha + \beta = 1$$

$$r \cdot \beta^r + r \cdot \alpha^r - r \cdot \beta = 14$$

$$\alpha\beta = -\frac{b}{a} \rightarrow \frac{b}{a} = -\frac{1}{r}$$

$$r \cdot \beta^r + r \cdot \alpha^r + r \cdot \beta^r - r \cdot \beta = 14$$

$$r \cdot [(1 + \beta)^r - r\alpha\beta] + r \cdot (\beta^r - \beta) = 14$$

$$r(1 + r \frac{b}{a}) + r \cdot \frac{b}{a} = 14$$

$$(1 - \beta)^r = (\alpha + \beta)^r - r\alpha\beta = 1 - \frac{1}{a} = \frac{r}{a} \rightarrow \alpha - \beta = \frac{r}{\sqrt{a}} = \frac{r\sqrt{a}}{a}$$

$$x^r - (a+1)x + a = 0$$

$$\rightarrow \alpha, \alpha_r = a$$

$$\alpha_1 + \alpha_r = a + 1$$

$$\alpha_1 + \alpha_r = r n, \alpha_1 \alpha_r = r n - 1$$

$$x^r - (ra+1)x + b = 0$$

$$\rightarrow y_1 + y_r = r n + 1$$

$$\alpha_1 + \alpha_r = a + 1 \rightarrow r n = a + 1 \rightarrow a = r n - 1$$

$$y_1 = r m, y_r = r m + 1$$

$$y_1 y_r = r m(r m + 1)$$

$$\alpha_1 \alpha_r = a \rightarrow r n - 1 = a \rightarrow a = r n - 1$$

$$\Rightarrow y_1 + y_r = r a + 1 = r x^r + 1 = 10 \rightarrow r m + r = 10$$

$$\Rightarrow r n - 1 = r n - 1 \Rightarrow r n - r n = 0 \rightarrow r n(r n - 1) = 0$$

$$\rightarrow r m = 1 \rightarrow m = r \rightarrow y_1 y_r = r x^r x^r = r^2$$

$$a = r n - 1 = r \rightarrow \alpha_1 = 1, \alpha_r = r \rightarrow 1 \cdot r^r = r^r = a \checkmark$$

$$y_1 = r, y_r = r \rightarrow \text{جواب} = r^2$$

$$y_1 y_r - \alpha_1 \alpha_r = r^2 - r^2 = 0 \rightarrow \text{جواب} = r$$

$$x^r + x - 1 - m^r = 0 \rightarrow \alpha + \beta = -1, \alpha\beta = -1 - m^r$$

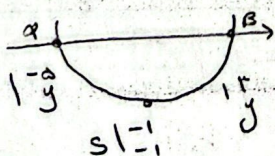
$$\alpha^r + \beta^r = ? \rightarrow (\alpha + \beta)^r - r\alpha\beta = (-1)^r - r(-1 - m^r) = 1 + r + r m^r = r + r m^r$$

$$m = 0 \rightarrow \text{جواب}$$

$$\min(\alpha^r + \beta^r) = r \rightarrow r + r \cdot 0 = r$$

$$A(r, y)$$

$$B(-a, y)$$



$$\frac{\alpha + \beta}{r} = \frac{-a + r}{r} = -1$$

$$\alpha + \beta = -r \rightarrow -\frac{r}{r} = -1$$

$$\alpha^r + \beta^r = a \rightarrow (\alpha + \beta)^r - r\alpha\beta = a \rightarrow \alpha\beta = -\frac{1}{r} \rightarrow y = a(\alpha^r + \beta^r - \frac{1}{r}) \Rightarrow 1 = a(1 - \frac{1}{r})$$

$$\rightarrow a = -\frac{r}{r-1} \rightarrow y = -\frac{r}{r-1}(\alpha^r + \beta^r - \frac{1}{r}) = y = \frac{1}{r}$$