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نام و نام خانوادگی ....ترنم اسلامی........ پاسخنامه تشریحی تکلیف شماره ...۱۹... کلاس .....A.......

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$3x^2 - ax + 4 = 0$

$\alpha = 3\beta$ ، $\alpha+\beta = \frac{a}{3}$ ، $\alpha\beta = \frac{4}{3}$ ۴

$3\beta+\beta = \frac{a}{3} \to 4\beta = \frac{a}{3} \to a = 12\beta \to a = \pm 2 \to 12(\pm\frac{2}{3}) = \pm 8 \to 8-(-8) = 16$

$3\beta(\beta) = \frac{4}{3} \to 3\beta^2 = \frac{4}{3} \to \beta^2 = \frac{4}{9} \to \beta = \pm\frac{2}{3}$

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$36x^2 - (m+14)x + 1 = 0$

$\frac{1}{\sqrt{r_1}} + \frac{1}{\sqrt{r_2}} = 5 \Rightarrow \frac{\sqrt{r_1}+\sqrt{r_2}}{\sqrt{r_1 r_2}} = \frac{\sqrt{r_1+r_2+2\sqrt{r_1r_2}}}{\sqrt{r_1r_2}} = \frac{\sqrt{S+\frac{1}{3}}}{\frac{1}{6}} = 5 \Rightarrow \sqrt{S+\frac{1}{3}} = \frac{5}{6}$

$(\sqrt{r_1}+\sqrt{r_2})^2 = r_1+r_2+2\sqrt{r_1r_2} \longrightarrow P = \frac{1}{36}$ ۴

$\to S = \frac{m+14}{36} = \frac{13}{36} \to \boxed{m=-1}$

$\Rightarrow S + \frac{1}{3} = \frac{25}{36} \to S = \frac{13}{36}$

$mx^2+3x+2=0 \to P = \frac{2}{m} = -2$

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$4x^3 + kx^2 - 9x - 2 = 0 \to x^2 - (\alpha+\beta)x + \alpha\beta = x^2 - (1)x + (-2) = x^2 - x - 2$ ۴

$\alpha+\beta = 1 \quad \to (x^2-x-2)(px+q) \to px^3 - px^2 + qx^2 - qx - 2px - 2q$

$\alpha\beta = -2 \quad \to px^3 + (q-p)x^2 + (-q-2p)x - 2q \xrightarrow{\text{تطابق}} p=4 ، q-p=k \to k = q-4$

معادله داده شده: $-q-2p = -9 \to -q-8 = -9 \to q=1$ ۴

$-2q \xrightarrow{\text{چک}} -2$ ✓

$\Rightarrow k = q-4 \to 1-4 = -3$

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$x^2+6x+a=0$ ، $A = \alpha\beta = \alpha(-6-\alpha) = -\alpha^2 - 6\alpha \to -\left(\frac{24+\sqrt{836+240\sqrt{2}}}{100}\right)^2 + \frac{144+6\sqrt{836+240\sqrt{2}}}{10}$

$a<\beta<0 \quad \to \boxed{A=1}$ ۴

$3a^2+2\beta^2 = 12\sqrt{2}+85 \to 3a^2 + 2\underbrace{(-6-\alpha)^2}_{a^2+12a+36} = 12\sqrt{2}+85 \Rightarrow 3a^2+2a^2+24a+72 = 12\sqrt{2}+85$

$\alpha+\beta = -6$ ، $\alpha\beta = A$

$\beta = -6-a$

$\Rightarrow 5a^2+24a+72-85-12\sqrt{2} = 0 \to 5a^2+24a-13-12\sqrt{2} = 0$

$\to a = \frac{-24\pm\sqrt{24^2-4\times5\times(-13-12\sqrt{2})}}{10} = \frac{-24-\sqrt{836+240\sqrt{2}}}{10}$ ، $\beta = \frac{-36+\sqrt{836+240\sqrt{2}}}{10}$

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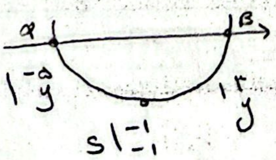
$x^4 - 7x^2 - 5 = 0 \xrightarrow{\text{ریشه}} +\sqrt{\frac{7+\sqrt{69}}{2}}$ ، $-\sqrt{\frac{7+\sqrt{69}}{2}} \xrightarrow{\text{مجموع}} \sqrt{\frac{7+\sqrt{69}}{2}} - \sqrt{\frac{7+\sqrt{69}}{2}} = 0$

$2P^2 - 3SP + 2S = ?$

$\Rightarrow \left(-\sqrt{\frac{7+\sqrt{69}}{2}}\right)\left(\sqrt{\frac{7+\sqrt{69}}{2}}\right) = -\frac{7+\sqrt{69}}{2} \Rightarrow \frac{-7-\sqrt{69}}{2}$ ۴

$\longrightarrow S=0$ ، $P = \frac{-7-\sqrt{69}}{2} \Rightarrow 2\left(\frac{-7-\sqrt{69}}{2}\right)^2 - 3(0)\left(\frac{-7-\sqrt{69}}{2}\right) + 2(0)$

$\Rightarrow \boxed{59+7\sqrt{69}}$

$2x^2 - ax + b = 0$ $2ax^2 + ax - 6 = 0$ $\left[\frac{ab}{4}\right] = \left[\frac{-6}{4}\right] = -2$ $\alpha+\beta = \alpha'+\beta'+1 \to \frac{a}{2} = \frac{-a}{2a} + 1 = \frac{1}{2} \to a=1 \to 2x^2+x-6=0 \to \alpha', \beta' = -2, \frac{3}{2} \to \alpha, \beta = -\frac{3}{2}, 2$ $\to \frac{b}{2} = \alpha\beta = -3 \to b=-6 \to \left[\frac{(1)(-6)}{4}\right] = -2$	6
$ax^2 - ax - b = 0 \to \alpha+\beta = 1$ $40\beta^2 + 20\alpha^2 - 20\beta = 17 \qquad \alpha\beta = -\frac{b}{a} \to \frac{b}{a} = -\frac{1}{20}$ $20\beta^2 + 20\alpha^2 + 20\beta^2 - 20\beta = 17$ $20\left[(\alpha+\beta)^2 - 2\alpha\beta\right] + 20(\beta^2-\beta) = 17$ $20\left(1 + 2\frac{b}{a}\right) + 20\frac{b}{a} = 17$ $(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta = 1 - \frac{1}{5} = \frac{4}{5} \to \alpha-\beta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$	7
$x^2 - (a+1)x + a = 0 \to x_1x_2 = a,\ x_1+x_2 = a+1 \to x_1 = 2n-1,\ x_2 = 2n+1$ $x_1+x_2 = 4n,\ x_1x_2 = 4n^2-1$ $x^2 - (3a+1)x + b = 0 \to y_1+y_2 = 4m+2$ $y_1 = 2m,\ y_2 = 2m+2 \qquad y_1y_2 = \underline{2m(2m+2)} = 4m(m+1)$ $x_1+x_2 = a+1 \to 4n = a+1 \to a = 4n-1$ $x_1x_2 = a \to 4n^2-1 = a \to a = 4n^2-1$ $\Rightarrow y_1+y_2 = 3a+1 = 3\times3+1 = 10 \to 4m+2 = 10$ $\Rightarrow 4n-1 = 4n^2-1 \Rightarrow 4n^2-4n = 0 \to 4n(n-1) = 0 \to \boxed{n=1}$ $\to 4m = 8 \to \boxed{m=2} \to y_1y_2 = 4\times2\times3 = 24$ $a = 4n-1 = 3 \to x_1 = 1,\ x_2 = 3 \to 1\times3 = 3$ ✓ $y_1 = 4,\ y_2 = 6 \to \boxed{\text{ضرب} = 24}$ $\Rightarrow \boxed{y_1y_2 - x_1x_2 = 24-3 = 21} \leftarrow \boxed{\text{ضرب} = 3}$	8
$x^2 + x - 1 - m^2 = 0 \to \alpha+\beta = -1,\ \alpha\beta = -1-m^2$ $\alpha^2+\beta^2 = ? \to (\alpha+\beta)^2 - 2\alpha\beta = (-1)^2 - 2(-1-m^2) = 1+2+2m^2 = 3+2m^2 \xrightarrow[\text{حذوف}]{m^2 \geq 0}$ تساوی $\leftarrow m=0$ $\min(\alpha^2+\beta^2) = 3 \to 3+2(0) = 3$	9
$A(3, y)$ $B(-5, y)$  $\frac{\alpha+\beta}{2} = \frac{-5+3}{2} = -1$ $\alpha+\beta = -2 \to -\frac{2}{2} = -1$ $\alpha^2+\beta^2 = 5 \to (\alpha+\beta)^2 - 2\alpha\beta = 5 \to \alpha\beta = -\frac{1}{2} \to y = a(x^2+2x-\frac{1}{2}) \Rightarrow 1 = a(1-2-\frac{1}{4})$ $\to a = -\frac{2}{4} \to y = -\frac{2}{4}(x^2+2x-\frac{1}{2}) \Rightarrow y = \frac{1}{4}$	10