

۱ $3x^2 - ax + 4 = 0$ ریشه ها: 3α و α

ضرب ریشه: $\frac{c}{a} = \frac{4}{3} = 3\alpha^2 \rightarrow \alpha^2 = \frac{4}{9} \rightarrow \alpha = \pm \frac{2}{3}$

ریشه ها: 2 و $\frac{2}{3}$ جمع ریشه ها: $\frac{1}{3} = -\frac{b}{a} = \frac{a}{3} \rightarrow \alpha = 1$ $\Delta = 44 - 48 > 0 \checkmark$

ریشه ها: -2 و $-\frac{2}{3}$ جمع ریشه ها: $-\frac{1}{3} = \frac{a}{3} \rightarrow \alpha = -1$ اختلاف مقدار: $1 - (-1) = 2$

$\Delta = 44 - 48 \checkmark$

۲ $4x^2 - (m+14)x + 1 = 0$ ریشه ها: α و β $\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \rightarrow \frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} = 5$

ضرب ریشه: $\frac{c}{a} = \frac{1}{4} = \alpha\beta \xrightarrow{\sqrt{\alpha\beta}} \sqrt{\alpha\beta} = \frac{1}{4}$ ①

① $\frac{\sqrt{\alpha} + \sqrt{\beta}}{\frac{1}{4}} = 5 \rightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{5}{4}$ $(\sqrt{\alpha} + \sqrt{\beta})^2 = \frac{25}{16} \rightarrow \alpha + \beta + 2\sqrt{\alpha\beta} = \frac{25}{16}$

$\Rightarrow \alpha + \beta = \frac{25}{16} - 2\sqrt{\alpha\beta} = \frac{25}{16} - \frac{1}{2} = \frac{17}{16} = -\frac{b}{a} = \frac{m+14}{4} \rightarrow m+14 = 17 \rightarrow m = 3$ $\Delta = (3)^2 - 4 \times 4 \times 1 > 0 \checkmark$

$4x^2 + 3x + 2 = 0 \rightarrow \Delta = 9 - 4(4)(2) < 0$ ریشه ندارد

۳ $4x^3 + 14x^2 - 9x - 2 = 0$ $\alpha + \beta = 1$ $\alpha\beta = -\frac{2}{4} \rightarrow \alpha = -\frac{2}{\beta}$

$\frac{-2}{\beta} + \beta = 1 \rightarrow \frac{-2 + \beta^2}{\beta} = 1 \rightarrow \beta^2 - \beta - 2 = 0 \rightarrow \beta = -1$ و 2

$\beta = -1 \xrightarrow{x=-1} 4(-1)^3 + 14(-1)^2 - 9(-1) - 2 = 0 \rightarrow 14 = -3$

$\beta = 2 \xrightarrow{x=2} 4(2)^3 + 14(2)^2 - 9(2) - 2 = 0 \rightarrow 14 = -3$

۴ $x^2 + 4x + a = 0 \rightarrow \alpha = \frac{-4 - \sqrt{16 - 4a}}{2} = -2 - \sqrt{4 - a} \rightarrow \alpha^2 = 18 - a + 4\sqrt{4 - a}$

$\beta = \frac{-4 + \sqrt{16 - 4a}}{2} = -2 + \sqrt{4 - a}$ $\left\{ \begin{array}{l} S = -\frac{b}{a} = -4 \\ P = \frac{c}{a} \end{array} \right.$

$2\alpha^2 + \alpha^2 + 2\beta^2 = 12\sqrt{4 - a} + 18 \rightarrow 2(\alpha^2 + \beta^2) + \alpha^2 = 12\sqrt{4 - a} + 18$

$2(34 - 2a) + 18 - a + 4\sqrt{4 - a} = 12\sqrt{4 - a} + 18 \rightarrow -5a + 4\sqrt{4 - a} = -5 + 4\sqrt{4 - a}$

$\Rightarrow \alpha = 1$ $\Delta = 34 - 4 > 0 \checkmark$

۵ $x^2 - 7x - 5 = 0 \rightarrow x^2 = t \rightarrow t^2 - 7t - 5 = 0 \rightarrow t = \frac{7 \pm \sqrt{49 + 20}}{2}$

$t = x^2$ پس t مساوی $\frac{7 - \sqrt{69}}{2}$ نمی تواند باشد چون منفی است در حالی که t باید مثبت باشد

$x^2 = \frac{7 + \sqrt{69}}{2} \rightarrow x = \pm \sqrt{\frac{7 + \sqrt{69}}{2}}$ صفر = جمع ریشه (S)

ضرب ریشه (P) $\rightarrow \frac{-7 - \sqrt{69}}{2}$ $2P^2 - 3SP + 2S = 2\left(\frac{-7 - \sqrt{69}}{2}\right)^2 - 3\left(\frac{7 + \sqrt{69}}{2}\right)\left(\frac{-7 - \sqrt{69}}{2}\right) + 2\left(\frac{7 + \sqrt{69}}{2}\right)$

$= \frac{118 + 14\sqrt{69}}{2} = 59 + 7\sqrt{69}$

$$2x^2 - ax + b = 0 \quad \begin{cases} \alpha + \frac{1}{\beta} \\ \beta + \frac{1}{\alpha} \end{cases} \rightarrow \text{جمع ریشه} = \alpha + \beta + 1 = S_1 = \frac{-b}{2a} = \frac{a}{2}$$

$$2ax^2 + ax - 4 = 0 \quad \begin{cases} \alpha \\ \beta \end{cases} \rightarrow \text{جمع ریشه} = \alpha + \beta = S_2 = \frac{-b}{2a} = \frac{-1}{2}$$

$$S_1 = S_2 + 1 \rightarrow \frac{a}{2} = \frac{-1}{2} + 1 \rightarrow \alpha = 1 \rightarrow 2x^2 - x + b = 0$$

$$2x^2 - x + b = 0 \rightarrow \text{ضرب ریشه} = \alpha\beta + \frac{\alpha}{\beta} + \frac{\beta}{\alpha} + \frac{1}{\beta} = -\frac{c}{a} = -\frac{4}{2} = -2$$

$$\left[\frac{+4 \times 1}{2} \right] = \left[\frac{+4}{2} \right] = 2$$

$$ax^2 - ax - b = 0 \quad S = \alpha + \beta = \frac{-b}{a} = \frac{a}{a} = 1 \quad P = \alpha\beta = \frac{c}{a} = \frac{-b}{a}$$

$$4\beta^2 + 2\alpha^2 - 2\beta = 17 \rightarrow 4\beta^2 + 1\beta^2 + 2\alpha^2 - 1\alpha^2 - 2\beta = 17 \rightarrow 5(\alpha^2 + \beta^2) - 1(\alpha^2 - \beta^2) - 2\beta = 17$$

$$\Rightarrow 5(1 + \frac{2b}{a}) - 1(\alpha - \beta) - 2\beta = 17 \rightarrow 5(1 + \frac{2b}{a}) - 1\alpha + 1\beta - 2\beta = 17$$

$$\rightarrow 5(1 + \frac{2b}{a}) - 1(\alpha + \beta) = 17 \rightarrow 5(1 + \frac{2b}{a}) = 27 \rightarrow 1 + \frac{2b}{a} = \frac{27}{5} \rightarrow \frac{b}{a} = \frac{1}{5}$$

$$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2 + 4ab}}{|a|} = \frac{|a|\sqrt{1 + \frac{4b}{a}}}{|a|} = \sqrt{1 - \frac{1}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$x^2 - (a+1)x + a = 0 \rightarrow \text{جمع ریشه: } 1 + a - a - 1 = 0 \Rightarrow \begin{cases} x_1 = 1 \\ x_2 = a \end{cases} \rightarrow \text{مقدار } a \text{ و } x_1, x_2 \text{ متوالی هستند} \rightarrow a = 3$$

$$x^2 - 4x + 3 = 0 \rightarrow P_1 = 3$$

$$x^2 - (3a+1)x + b = 0 \rightarrow x^2 - 10x + b = 0$$

$$S = \alpha + \beta = 10 \rightarrow \alpha = \beta + 2 \rightarrow \beta + 2 + \beta = 10 \rightarrow 2\beta = 8 \rightarrow \beta = 4 \rightarrow \alpha = 6$$

$$P = \alpha\beta = 24 = b \rightarrow P_2 = 24$$

$$x^2 + x - 1 - m^2 = 0$$

$$\Delta \geq 0 \rightarrow b^2 - 4ac \geq 0 \Rightarrow 1 + 4 + 4m^2 \geq 0 \rightarrow 4m^2 + 5 \geq 0 \rightarrow \text{همواره برقرار}$$

$$\alpha^2 + \beta^2 = S^2 - 2P = 1 + 2m^2 + 2 = 2m^2 + 3 \rightarrow \text{کمترین مقدار این عبارت به ازای کمترین مقدار } m^2 \text{ است}$$

$$S = \frac{-b}{a} \quad P = \frac{c}{a} \rightarrow \text{مقدار } m^2 \text{ برقرار است که چون همواره نامنتی است پس کمترین مقدار آن صفر است}$$

$$\hookrightarrow m^2 = 0 \rightarrow \alpha^2 + \beta^2 = 3$$

$$\frac{-b}{2a} = -1 \rightarrow b = 2a \quad \leftarrow x_5 = \frac{x_1 + x_2}{2} \leftarrow \text{چون } A \text{ و } B \text{ یکی است پس}$$

$$y = ax^2 + bx + c \xrightarrow{x=-1} a - b + c = 1 \quad \frac{b=2a}{b=2a} \rightarrow a - 2a + c = 1 \rightarrow c = 1 + a$$

$$\alpha^2 + \beta^2 = S^2 - 2P = \frac{b^2}{a^2} - \frac{2c}{a} \xrightarrow{\frac{c=1+a}{b=2a}} \frac{4a^2}{a^2} - \frac{2(1+a)}{a} = 4 - \frac{2+2a}{a} = 2 - \frac{2a}{a} = 0$$

$$S = \frac{-b}{2a} \quad P = \frac{c}{a}$$

$$\rightarrow -a = 2 + 2a \rightarrow a = \frac{-2}{3} \rightarrow b = \frac{-4}{3} \quad \leftarrow \text{مقدار از مبدا} \quad c = \frac{1}{3}$$