

$$3x^2 - ax + 1 = 0 : \alpha, \beta \Rightarrow 3\alpha^2 = \frac{a}{3} \Rightarrow \alpha^2 = \frac{a}{9} \Rightarrow \alpha = \pm \frac{\sqrt{a}}{3}$$

$$\Rightarrow \frac{a}{9} = \alpha^2 \Rightarrow \alpha \times 12 = a \begin{cases} \alpha_1 = \frac{\sqrt{a}}{3} \times 12 = 4\sqrt{a} \\ \alpha_2 = -\frac{\sqrt{a}}{3} \times 12 = -4\sqrt{a} \end{cases} \Rightarrow \alpha_1 - \alpha_2 = 4\sqrt{a} - (-4\sqrt{a}) = 8\sqrt{a}$$

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$$x^2 - (m+1)x + 1 = 0 \Rightarrow \alpha, \beta : \frac{1}{\alpha} + \frac{1}{\beta} = m+1 : \frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha}\sqrt{\beta}} = m+1 \Rightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{m+1}{\sqrt{\alpha}\sqrt{\beta}}$$

$$\Rightarrow \alpha + \beta = \frac{1}{\sqrt{\alpha}\sqrt{\beta}} \Rightarrow \frac{m+1}{\sqrt{\alpha}\sqrt{\beta}} = \frac{1}{\sqrt{\alpha}\sqrt{\beta}} \Rightarrow m+1 = 1 \Rightarrow m = 0$$

$$\Rightarrow m\alpha^2 + \alpha + 1 = 0 : 1\alpha^2 + \alpha + 1 = 0 : \Delta = 1 - 4 = -3$$

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$$x^2 + kx - a = 0 : \alpha, \beta \mid \alpha + \beta = 1, \alpha\beta = -a \Rightarrow k = ?$$

$$\Rightarrow (\alpha + \beta)^2 = \alpha^2 + \beta^2 + 2\alpha\beta \Rightarrow 1 = \alpha^2 + \beta^2 - 2a \Rightarrow \alpha^2 + \beta^2 = 1 + 2a$$

$$\Rightarrow (\alpha - \beta)^2 = \alpha^2 + \beta^2 - 2\alpha\beta = 1 + 2a + 2a = 1 + 4a \Rightarrow \alpha - \beta = \pm \sqrt{1 + 4a}$$

$$\Rightarrow \alpha + \beta = 1, \alpha - \beta = \pm \sqrt{1 + 4a} \Rightarrow \alpha = \frac{1 \pm \sqrt{1 + 4a}}{2}, \beta = \frac{1 \mp \sqrt{1 + 4a}}{2}$$

$$\Rightarrow \alpha\beta = -a \Rightarrow \frac{(1 + \sqrt{1 + 4a})(1 - \sqrt{1 + 4a})}{4} = -a \Rightarrow \frac{1 - (1 + 4a)}{4} = -a \Rightarrow \frac{-4a}{4} = -a \Rightarrow -a = -a$$

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$$x^2 + 2x + a = 0 : \alpha < \beta < 0, \alpha^2 + \beta^2 = 14\sqrt{2} + 10 : a = ?$$

$$\Rightarrow \alpha^2 + \beta^2 + 2\alpha\beta = 14\sqrt{2} + 10 \Rightarrow \alpha^2 + \beta^2 = 14\sqrt{2} + 10 - 2\alpha\beta$$

$$\Rightarrow \alpha^2 + \beta^2 = 14\sqrt{2} + 10 - 2(-a) = 14\sqrt{2} + 10 + 2a$$

$$\Rightarrow 14\sqrt{2} + 10 + 2a = 14\sqrt{2} + 10 \Rightarrow 2a = 0 \Rightarrow a = 0$$

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$$x^2 - 2x - a = 0 : \Delta = 4 + 4a = 4(1 + a) \Rightarrow \Delta = 4(1 + a) \Rightarrow \sqrt{\Delta} = 2\sqrt{1 + a}$$

$$\Rightarrow x = \frac{2 \pm 2\sqrt{1 + a}}{2} = 1 \pm \sqrt{1 + a}$$

$$\Rightarrow x_1 = 1 + \sqrt{1 + a}, x_2 = 1 - \sqrt{1 + a}$$

$$\Rightarrow x_1^2 - 2x_1 - a = 0 \Rightarrow (1 + \sqrt{1 + a})^2 - 2(1 + \sqrt{1 + a}) - a = 0$$

$$\Rightarrow 1 + 2\sqrt{1 + a} + 1 + a - 2 - 2\sqrt{1 + a} - a = 0 \Rightarrow 0 = 0$$

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$$r n^r - a n + b = 0 : \alpha + \frac{1}{r}, \beta + \frac{1}{r} \Rightarrow \left[\frac{ab}{r} \right] = ?$$

$$r a n^r + a n - r = 0 \rightarrow \alpha, \beta : s = \frac{-a}{ra} = -\frac{1}{r}, P = \frac{-r}{ra} = -\frac{r}{a}$$

Q1) $s = \alpha + \beta + 1 = \frac{a}{r} \Rightarrow \frac{a}{r} = \frac{1}{r} \Rightarrow \boxed{a=1}$, $P = \alpha + \beta + \frac{1}{r} (\alpha + \beta) + \frac{1}{r} = \frac{b}{r} : -r + \frac{1}{r} + \frac{1}{r} = \frac{b}{r}$

$$\Rightarrow \underline{b=4} \Rightarrow \left[\frac{ab}{r} \right] = \left[\frac{4}{r} \right] = \left[\frac{r}{r} \right] = 1$$

$$a n^r - a n - b = 0 : \epsilon, \beta + r, \alpha^r - r, \beta = 1 \Rightarrow r, (\beta^r + \alpha^r) + r, (\beta^r - \beta) = 1$$

$$(L) a(\beta^r - \beta) = b : \beta^r - \beta = \frac{b}{a} \Rightarrow r, \left(\frac{b}{a} \right) + r, (s^r - rP) = 1$$

$$1 + \frac{r b}{a} = \frac{a + r b}{a}$$

$$\Rightarrow \frac{r, b}{a} + \frac{r, a + \epsilon, b}{a} = \frac{r, b + r, a}{a} = 1 \Rightarrow r, b = -r, a : \boxed{a = -r, b}$$

$$\Rightarrow |\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{\epsilon, b^r - 4, a, b^r}}{| -r, b |} = \frac{\sqrt{r, r, |b|}}{r, |b|} = \frac{1 \sqrt{a}}{r} = \frac{r \sqrt{a}}{a}$$

$$n^r - (a+1)n + a = 0 : \alpha + \beta + 1 = 0 \Rightarrow n=1, a = 0 \Rightarrow 1, r = 0 \Rightarrow \boxed{a=r}$$

$$n^r - (ra+1)n + b = 0 : \alpha + \beta = r : n^r - 1, n + b = 0 \Rightarrow \begin{cases} s = 1 \\ P = b \end{cases} \Rightarrow \alpha = \beta + r$$

$$\Rightarrow s = r + 1 = \epsilon + k + r \Rightarrow r, a = \epsilon + 1 : r, k + r = k + 1 : 1, k, a = r, k + 1$$

$$\Rightarrow P_r - P_1 = r\epsilon - r = r_1$$

$$\Rightarrow P_r = b = d\beta = \epsilon \times r = r_\epsilon$$

$$n^r + n + (-m^r - 1) = 0 : \alpha, \beta \Rightarrow \text{Min} : \alpha^r + \beta^r = (s)^r - rP : 1 - rP$$

$$\hookrightarrow s = -1, P = -m^r - 1 \Rightarrow 1 + r m^r + r = r m^r + r \Rightarrow \underline{r} \rightarrow \text{مطلوب}$$

$$y_s = \frac{\Delta}{ca} = 1, \begin{cases} A(r, y) \\ B(-a, y) \end{cases} \Rightarrow n_s = \frac{r-a}{r} = -1 \Rightarrow \begin{cases} n_s = -1 \\ y_s = 1 \end{cases} \Rightarrow y = a(n+1)^r + 1 = \underline{a n^r + r a n + a + 1}$$

$$\Rightarrow \alpha^r + \beta^r = 0 \Rightarrow \left(\frac{-ra}{a} \right)^r - r \left(\frac{a+1}{a} \right) = 0 = -\frac{r-r}{a} = 1 \rightarrow -ra - r = a$$

$$\Rightarrow \boxed{a = \frac{-r}{r}}$$

$$\Rightarrow y = \frac{-r}{r} n^r - \frac{r}{r} n + \left(\frac{-r}{r} + 1 \right)^r \Rightarrow n = 0 : \underline{y = \frac{1}{r}}$$