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★ خارجی تألیف ۱۹ ، یازدهم دختر A ★

۱) $3x^2 - ax + 4 = 0 \to$ ریشه‌ها : $\alpha , 3\alpha \to P = 3\alpha^2 = \frac{4}{3} \Rightarrow \alpha = \pm \frac{2}{3}$

① : $\alpha = \frac{2}{3} \to 3\left(\frac{4}{9}\right) - \frac{2a}{3} + 4 = 0 \to a_1 = 8$ $\quad \to a_1 - a_2 = 16$

② : $\alpha = -\frac{2}{3} \to 3\left(\frac{4}{9}\right) + \frac{2a}{3} + 4 = 0 \to a_2 = -8$

۲) $36x^2 - (m+14)x + 1 = 0 \quad \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5$

$*\ (\sqrt{\alpha} + \sqrt{\beta})^2 = \underbrace{\alpha + \beta}_{S} + 2\underbrace{\sqrt{\alpha\beta}}_{P} = \frac{m+14}{36} + \frac{2}{6} = \frac{m+26}{36} \Rightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{\sqrt{m+26}}{6}$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} = \frac{\frac{\sqrt{m+26}}{6}}{\frac{1}{6}} = \sqrt{m+26} = 5 \to m + 26 = 25$

$\Rightarrow m = -1$

$\to -1x^2 + 3x + 2 = 0 \to$ حاصلضرب ریشه‌ها $P = \frac{2}{-1}$

۳) $S = 1 , P = \alpha\beta = -2 \to \alpha , \frac{-2}{\alpha} \to \alpha - \frac{2}{\alpha} = \frac{\alpha^2 - 2}{\alpha} = 1$

$\to \alpha^2 - 2 - \alpha = 0 \to (\alpha - 2)(\alpha + 1) \to$ ریشه‌ها : $2 , -1$

$x = -1 \to -4 + k + 9 - 2 = 0 \to k = -3$

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(۴ $x^2 + 4x + a = 0 \rightarrow 3\alpha^2 + 2\beta^2 = 2\alpha^2 + \alpha^2 + 2\beta^2 - \beta^2 =$

$2(\alpha^2 + \beta^2) + \alpha^2 - \beta^2 = 2(s^2 - 2p) + \frac{1}{2}s\left(\frac{\sqrt{\Delta}}{|a|}\right) =$

$\frac{\omega}{2}(36 - 2a) - 4\sqrt{9-a} = 90 - 5a + 4\sqrt{9-a} = 12\sqrt{2} + 8\omega$

$90 - 8\omega - 12\sqrt{2} - 5a = 4\sqrt{9-a}$

$-50a + 120a\sqrt{2} + 25a^2 - 120\sqrt{2} = 36(9-a)$

$\rightarrow 25a^2 - 120\sqrt{2} + (120\sqrt{2} - 50)a = 11 - 36a \rightarrow a = 1$

(۷ $ax^2 - ax - b = 0 \rightarrow S = 1 \rightarrow \alpha + \beta = 1 \rightarrow \beta = 1 - \alpha$

حالت اول: $40(1-\alpha)^2 + 20\alpha^2 - 20(1-\alpha) = 17$

$40(1 - 2\alpha + \alpha^2) + 20\alpha^2 - 20 + 20\alpha = 17$

$40 - 80\alpha + 40\alpha^2 + 20\alpha^2 - 20 + 20\alpha = 17 \rightarrow 60\alpha^2 - 60\alpha + 20 = 17$

$\rightarrow 60\alpha^2 - 60\alpha + 3 = 0 \xrightarrow{\div 3} 20\alpha^2 - 20\alpha + 1 = 0$

چون α ریشه معادله قبلیه س:

$|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{400 - 80}}{20} = \frac{8\sqrt{5}}{20} = \frac{2\sqrt{5}}{5}$

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$x^4 - 7x^2 - 5 = 0 \quad \xrightarrow{x^2 = t} \quad t^2 - 7t - 5 = 0$ (a)

$\Delta = 49 - (4 \times 1 \times -5) = 49 + 20 = 69$

$\rightarrow t = \frac{7 \pm \sqrt{69}}{2} \rightarrow x^2 = \frac{7 + \sqrt{69}}{2}$

$\rightarrow x = \pm\sqrt{\frac{7+\sqrt{69}}{2}} \rightsquigarrow S = 0$

$\rightarrow p = \frac{-(7+\sqrt{69})}{2}$

$2p^2 - 3p + 2 = 2p^2 = 2\left(\frac{49 + 69 + 14\sqrt{69}}{4}\right) = \frac{118 + 14\sqrt{69}}{2}$

$= 59 + 7\sqrt{69}$

(b)

$2a^2 - 9a + b = 0 \rightarrow S_1 = 9/2 \quad P_1 = b/2 \rightarrow \alpha + \beta = 9/2$

$2a^2 + 9a - 2 = 0 \rightarrow S_2 = -9/2 \quad P_2 = \frac{-2}{2} \rightarrow \alpha + \beta + 1 = -1/2$

$\rightarrow S_{\alpha+1} = S_2 \rightarrow 9/2 + 1 = -1/2 \rightarrow a = -14$

$2a^2 + 3a + b = 0$

$-2a^2 - 3a - 2 = 0 \rightarrow P = (\alpha + 1/2)(\beta + 1/2) = \alpha\beta + \frac{\alpha+\beta}{2} + 1/4$

$= \frac{b}{2} - \frac{3}{4} + \frac{1}{4} = 1 \Rightarrow b = 3^0 \Rightarrow \left[\frac{9b^{-9}}{4}\right] = (-3)$

۷)

$x^2+x-1-m^2=0 \to S=-1 \to \alpha^2+\beta^2=S^2-2P=1+2+2m^2$

$\to P=-(1+m^2)$

کمترین مقدار: $m=0 \Rightarrow \boxed{S^2-2P=3}$

۸)

$x^2-(a+1)x+a=0 \to \alpha, \alpha+2 \to S=a+1=2\alpha+2$

$\to P=a=2\alpha+\alpha^2 \xrightarrow{+1} 2\alpha+\alpha^2+1=2\alpha+2$

$\alpha^2=1 \to \alpha=1 \Rightarrow \boxed{a=3}$

$x^2-10x+b=0 \to \beta, \beta+2$

طبق متن x

$\to P=b=2\beta+\beta^2,\ S=10=2\beta+2$

$\boxed{\beta=4 \Rightarrow b=24}$

$b-a=24-3=\boxed{21}$

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۱۰) راس $y = 1$, $x = \frac{3-5}{2} = x_s = \frac{-b}{2a}$ ← $A(3,y)$ $B(-5,y)$

$(-1,1) \rightarrow y = a(x+1)^2 + 1 = ax^2 + 2ax + a + 1$ *

$\Delta = \sqrt{-4a} \Rightarrow \alpha = \frac{-2a \pm \sqrt{-4a}}{2a} =$ $\nearrow -1 - \sqrt{-1/a}$, $\searrow -1 + \sqrt{-1/a}$ $\rightarrow P =$, $S = -2$

$\alpha^2 + \beta^2 = S^2 - 2P = 4 - 2(1 - 1/a) = 5 \rightarrow -2/a = 3$

$\rightarrow a = -2/3$

* $\rightarrow y = -2/3 (x+1)^2 + 1 \xrightarrow{x=0}$ $\boxed{y = 1/3}$