$3x^2 - 9x + 4 = 0$ $\quad \alpha + 2\alpha = 3\alpha = \frac{9}{3}$

نسا صادقی

①

$4 \times \frac{2}{3} = \frac{a}{3} \longrightarrow a = 8$ $\quad 8 - (-8) = 16$

$4 \times -\frac{2}{3} = \frac{a}{3} \longrightarrow a = -8$

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②

$36x^2 - (m+14)x + 1 = 0$ $\quad \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5$ $\quad mx^2 + 2x + 2 = 0$

$\boxed{\frac{2}{m} = \frac{2}{-1} = -2}$

$\frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} = 5 \longrightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{5}{6} \longrightarrow \alpha + \beta + 2\sqrt{\alpha\beta} = \frac{25}{36}$

$\frac{m+14}{36} + \frac{2}{6} = \frac{25}{36}$

$\alpha\beta = \frac{c}{a} = \frac{1}{36}$ $\quad \alpha + \beta = -\frac{b}{a} = \frac{m+14}{36} \longrightarrow m + 14 + 12 = 25$

$m = -1$

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③

$2x^3 + kx^2 - 9x - 2 = 0$

$\alpha + \beta = 1$

$\alpha\beta = -2 \longrightarrow \beta = \frac{-2}{\alpha}$

$\alpha + \frac{-2}{\alpha} = 1 \longrightarrow \frac{\alpha^2 - 2}{\alpha} = 1 \longrightarrow \alpha^2 - \alpha - 2 = 0$

$(\alpha - 2)(\alpha + 1) = 0 \longrightarrow \alpha = 2, \ \beta = -1$

$x = 2 \longrightarrow 32 + 4k - 18 - 2 = 0 \longrightarrow 4k = -12 \longrightarrow k = -3$

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④

$x^2 + 6x + a = 0$ $\quad a < \beta < 0$ $\quad 3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 85$

$\frac{5}{2}(\alpha^2 + \beta^2) + \frac{1}{2}(\alpha^2 - \beta^2) \longrightarrow \alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta \longrightarrow 36 - 2\alpha\beta$

$\alpha^2 - \beta^2 = (\alpha+\beta)(\alpha-\beta) = -6(\sqrt{5})$ $\quad 12\sqrt{2} + 85 = \frac{5}{2}(36 - 2\alpha\beta) + \frac{1}{2}(-6\sqrt{5})$

$24\sqrt{2} + 170 = 5(36 - 2\alpha\beta) - 6\sqrt{5} \longrightarrow 24\sqrt{2} = -6\sqrt{5} \longrightarrow \sqrt{5} = -4\sqrt{2}$ $\quad \boxed{a = 1}$

$\frac{170}{5} = 36 - 2\alpha\beta \longrightarrow 34 - 36 = 2\alpha\beta \longrightarrow 2\alpha\beta = 2 \longrightarrow \alpha\beta = 1$ $\quad \frac{c}{a} = \frac{a}{1} = 1$

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⑤

$x^4 - 7x^2 - 5 = 0 \longrightarrow t^2 - 7t - 5 = 0$ $\quad t = \frac{7 \pm \sqrt{49 + 20}}{2}$ $\quad t > 0$

$x^2 = t$ $\quad t = \frac{7 + \sqrt{69}}{2}$

$x^2 = \frac{7 + \sqrt{69}}{2} = x = \pm\sqrt{\frac{7 + \sqrt{69}}{2}} \longrightarrow S = \sqrt{\frac{7 + \sqrt{69}}{2}} - \sqrt{\frac{7 + \sqrt{69}}{2}} = 0$

$P = \frac{-7 - 69}{2}$

$2P^2 - \cancel{3S}P + \cancel{4S} = 2P^2 = 2 \times \left(\frac{-7 - \sqrt{69}}{2}\right)^2 \longrightarrow \frac{49 + 69 + 14\sqrt{69}}{2} = \frac{118 + 14\sqrt{69}}{2}$

$59 + 7\sqrt{69}$

$\to c = a - 4$ $\quad 4a = 1 \to \sqrt{b^2 - 4ac}$

(6)

$2x^2 - ax + b = 0 \qquad \alpha, \beta \to S = \frac{-b}{a} = \frac{a}{2}, \; P = \frac{c}{a} = \frac{b}{2}$

$2ax^2 + ax - 6 = 0 \qquad \alpha + \frac{1}{2}, \; \beta + \frac{1}{2} \to S = -\frac{b}{a} = \frac{-a}{2a} = -\frac{1}{2}$

$P' = \frac{-6}{2a} = -\frac{3}{a}$

$S' = \alpha + \frac{1}{2} + \beta + \frac{1}{2} \to \alpha + \beta + 1 = S + 1 \to -\frac{1}{2} = \frac{a}{2} + 1$

$a = -3 \leftarrow \frac{a}{2} = -\frac{3}{2}$

$P' = (\alpha + \frac{1}{2})(\beta + \frac{1}{2}) = \alpha\beta + \frac{1}{2}(\alpha + \beta) + \frac{1}{4} = \frac{b}{2} + \frac{1}{2} \times -\frac{3}{2} + \frac{1}{4} = \frac{-3}{-3} = 1$

$\left[\frac{ab}{4}\right] = \left[\frac{-9}{4}\right] = -3$

(7)

$a\alpha^2 - a\alpha - b = 0 \to a\alpha^2 = a\alpha + b \to \alpha^2 = \frac{a\alpha + b}{a}$

$a\beta^2 - \alpha\beta - b = 0 \to a\beta^2 = \alpha\beta + b = \beta^2 = \frac{a\beta^2 + b}{a}$

$\left(\frac{4}{20} \times \frac{a\beta + b}{a} + \frac{2}{20} \times \frac{a\alpha + b}{a} - 20\beta = 17\right) \xrightarrow{\times a} 40a\beta + 40b + 20a\alpha + 20b - 20\beta a = 17a \to$

$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2 + 4 \times a \times -\frac{3}{40}a}}{|a|} = \frac{\sqrt{\frac{7}{10}a^2}}{|a|} = \sqrt{\frac{7}{10}}$

(8)

$x^2 - (a+1)x + a = 2 \to \alpha, \alpha + 2 \to S = a + 1 = \alpha + \alpha + 2 = 2\alpha + 2$

$x^2 - (3a+1)x + b = 0 \to \beta, \beta + 2 \qquad P = a = \alpha(\alpha + 2) = \alpha^2 + 2\alpha$

$S = 10 = \beta + \beta + 2 \to 2\beta = 8 \to \beta = 4 \qquad \alpha^2 + 2\alpha + 1 = 2\alpha + 2 \to \alpha^2 = 1 \to \alpha = \pm 1$

$b = 4 \times 6 = 24 \qquad a = 1 \times 3 = 3 \qquad \alpha = 1 \leftarrow$

$24 - 3 = 21$

(9)

$x^2 + x - 1 - m^2 = 0 \qquad \alpha^2 + \beta^2 = S^2 - 2P = (-1)^2 - 2(-1 - m^2) = 1 + 2 + 2m^2 = 2m^2 + 3$

0 و $m^2 \to 3$

$S = \frac{-b}{a} = -1$

$P = \frac{c}{a} = -1 - m^2$

(۱۰

$A(۳, y)$
$B(-۵, y)$

$x_s = \dfrac{-۵+۳}{۲} = -۱ \qquad y_s = ۱$

$\dfrac{-b}{۲a} = -۱ \longrightarrow b = ۲a \qquad \dfrac{\sqrt{\Delta}}{۴a} = ۱ \longrightarrow \sqrt{b^۲ - ۴ac} = ۴a \rightarrow b^۲ - ۴ac = ۱۶a$

$a - c = ۴ \longrightarrow c = a - ۴ \qquad y = ax^۲ + ۲ax + a - ۴$

$S = \dfrac{-۲a}{a} = -۲$

$P = \dfrac{a-۴}{a}$

$\alpha^۲ + \beta^۲ = S^۲ - ۲P = (-۲)^۲ - ۲ \times \left(\dfrac{a-۴}{a}\right) = ۴ - ۲\left(۱ - \dfrac{۴}{a}\right) = ۴ - ۲ + \dfrac{۸}{a} = ۵$

$\dfrac{۸}{a} = ۳ \longrightarrow a = \dfrac{۸}{۳} \longrightarrow c = a - ۴ = \dfrac{۸}{۳} - ۴ = \dfrac{-۴}{۳}$