ماهک امیرمعتدلی ←

۱ ←

$3x^2 - ax + 4 = 0 \qquad \alpha + 3\alpha = 4\alpha = \frac{a}{3}$

$\alpha \times 3\alpha = 3\alpha^2 = \frac{4}{3} \longrightarrow \alpha^2 = \frac{4}{9} \longrightarrow a = \pm \frac{2}{3}$

$\left.\begin{array}{l} 4 \times \frac{2}{3} = \frac{a}{3} \Rightarrow a = 8 \\ 4 \times -\frac{2}{3} = \frac{a}{3} \Rightarrow a = -8 \end{array}\right\} \longrightarrow 8 - (-8) = \boxed{16}$

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۲ ←

$36x^2 - (m+14)x + 1 = 0 \qquad \frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5$

$mx^2 + 3x + 2 = 0 \longrightarrow \frac{2}{m} = ?$

$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\alpha} + \sqrt{\beta}}{\underbrace{\sqrt{\alpha\beta}}_{1/6}} = 5 \longrightarrow \sqrt{\alpha} + \sqrt{\beta} = \frac{5}{6} \xrightarrow{\text{توان ۲}}$

$\alpha + \beta + 2\sqrt{\alpha\beta} = \frac{25}{36} \longrightarrow \frac{m+14}{36} + \frac{2}{6} = \frac{25}{36} \longrightarrow$

$\alpha\beta = \frac{c}{a} = \frac{1}{36}$

$\alpha + \beta = \frac{-b}{a} = \frac{m+14}{36} \qquad m + 14 + 12 = 25 \longrightarrow m = -1$

$\frac{2}{m} = \frac{2}{-1} = \boxed{-2}$

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۳ ←

$4x^3 + kx^2 - 9x - 2 = 0$

$\left.\begin{array}{l} \alpha + \beta = 1 \\ \alpha\beta = -2 \Rightarrow \beta = \frac{-2}{\alpha} \end{array}\right\} \Rightarrow \alpha + \frac{-2}{\alpha} = 1 \Rightarrow \frac{\alpha^2 - 2}{\alpha} = 1 \Rightarrow \alpha^2 - \alpha - 2 = 0$

$(\alpha - ۲)(\alpha + ۱) = ۰ \longrightarrow \alpha = ۲ , \beta = -۱$

$\xrightarrow{x = ۲} ۳۲ + ۴k - ۱۸ - ۲ = ۰ \Rightarrow ۴k = ۱۲ \Rightarrow \boxed{k = -۳}$

$\alpha + \beta = -۶ , \alpha\beta = a$ ← ۴

$a < \beta < ۰$

$۳\alpha^۲ + ۲\beta^۲ = ۱۲\sqrt{۲} + ۸۵ \rightarrow$

$\frac{۵}{۲}(\alpha^۲ + \beta^۲) + \frac{۱}{۲}(\alpha^۲ - \beta^۲)$

$\alpha^۲ + \beta^۲ = (\alpha + \beta)^۲ - ۲\alpha\beta \longrightarrow ۳۶ - ۲\alpha\beta$

$\alpha^۲ - \beta^۲ = (\alpha + \beta)(\alpha - \beta) \longrightarrow -۶(\sqrt{\Delta})$

$۱۲\sqrt{۲} + ۸۵ = \frac{۵}{۲}(۳۶ - ۲\alpha\beta) + \frac{۱}{۲}(-۶\sqrt{\Delta})$

$۲۴\sqrt{۲} + ۱۷۰ = ۵(۳۶ - ۲\alpha\beta) - ۶\sqrt{\Delta}$

$\rightarrow ۲۴\sqrt{۲} = -۶\sqrt{\Delta} \longrightarrow \sqrt{\Delta} = -۴\sqrt{۲}$

$\frac{۱۷۰}{۵} = ۳۶ - ۲\alpha\beta \longrightarrow ۳۴ - ۳۶ = -۲\alpha\beta \Rightarrow$

$۲\alpha\beta = ۲ \longrightarrow \alpha\beta = ۱ \Rightarrow \frac{a}{a} = \frac{a}{۱} = ۱ \Rightarrow$

$\boxed{a = ۱}$

$$x^4 - 7x^2 - 5 = 0 \xrightarrow{x^2 = t} t^2 - 7t + 5 = 0$$ ← 5

$$t = \frac{7 \pm \sqrt{49 + 20}}{2} \xrightarrow{t > 0} t = \frac{7 + \sqrt{69}}{2}$$

$$\Rightarrow x^2 = \frac{7 + \sqrt{69}}{2} \Rightarrow x = \pm\sqrt{\frac{7 + \sqrt{69}}{2}}$$

$$S = \sqrt{\frac{7 + \sqrt{69}}{2}} - \sqrt{\frac{7 + \sqrt{69}}{2}} = 0$$

$$P = \frac{-7 - \sqrt{69}}{2}$$

$$2P^2 - \cancel{3S}P + \cancel{4S} = 2P^2 = 2 \times \left(\frac{-7 - \sqrt{69}}{2}\right)^2 =$$

$$\frac{118 + 14\sqrt{69}}{2} = \boxed{59 + 7\sqrt{69}}$$

$$2x^2 - ax + b = 0 \quad \alpha, \beta \rightarrow S = a/2$$ ← 6

$$P = b/2$$

$$2ax^2 + ax - 6 = 0 \quad \alpha + \frac{1}{2}, \beta + \frac{1}{2} \rightarrow S' = \frac{-a}{2a} = -1/2$$

$$P' = -\frac{6}{2a} = -\frac{3}{a}$$

$$S' = \alpha + \beta + 1 \Rightarrow -\frac{1}{2} = a/2 + 1 \rightarrow a = -3$$

$P' = (\alpha+\frac{1}{2})(\beta+\frac{1}{2}) = \alpha\beta + \frac{1}{2}(\alpha+\beta)+\frac{1}{4} =$

$\frac{b}{2} - \frac{3}{4} + \frac{1}{4} = \frac{-3}{a} = 1 \Longrightarrow \frac{b}{2} = \frac{3}{2} \longrightarrow b = 3$

$a = -3$

$[\frac{ab}{4}] = [-\frac{9}{4}] = \boxed{-3}$

$ax^2 - ax - b = 0 \qquad 40\beta^2 + 20\alpha^2 - 20\alpha = 17$

$\hookrightarrow x^2 = \frac{ax+b}{2} \longrightarrow \beta^2 = \frac{a\beta+b}{2}$

$40 \times \frac{\alpha\beta+b}{a} + 20 \times \frac{a\alpha+b}{a} - 20\beta = 17 \xrightarrow{\times\alpha}$

$40\alpha\beta + 40b + 20a\alpha + 20b - 20\beta\alpha = 17\alpha$

$\Longrightarrow 20a + 40b = 17a \Longrightarrow 40b = -3a \longrightarrow b = \frac{-3}{40}a$

اختلاف ریشه‌ها $\longrightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{a^2 + 4a \times \frac{-3}{40}a}}{|a|} = \frac{\sqrt{\frac{7}{10}a^2}}{|a|}$

$\boxed{= \sqrt{\frac{7}{10}}}$

$x^2-(a+1)x+a=0 \longrightarrow \alpha , \alpha+2$ ۸ ←

$\longrightarrow S = a+1 = 2\alpha+2$

$P = a = \alpha^2+2\alpha$

$\Longrightarrow \alpha^2+2\alpha+1 = 2\alpha+2 \longrightarrow \alpha^2 = 1 \longrightarrow \alpha = \pm 1$

$\xrightarrow{\alpha \text{ عدد طبیعی است}} a = 1\times 3 = 3$

$x^2-(\cancel{3a+1}^{10})x+b=0 \longrightarrow \beta , \beta+2$

$S = 10 = 2\beta+2 \longrightarrow \beta = 4$

$b = 4\times 6 = 24$

$b-a = 24-3 = \boxed{21}$

$x^2+x-1-m^2=0$ ۹ ←

$\alpha+\beta = \frac{-b}{a} = -1$

$\alpha\beta = -1-m^2$

$\longrightarrow \alpha^2+\beta^2 = (\alpha+\beta)^2-2\alpha\beta = 1-2(-1-m^2) = 2m^2+3 \xrightarrow{m^2\geq 0} \boxed{\min = 3}$

$A(3,y)$ $B(-5,y)$ $\xrightarrow{\text{هم عرض اند}}$ ۱۰ ←

$$x_s = \frac{-5+3}{2} = -1 \quad , \quad y_s = 1$$

$$y - 1 = a(x+1)^2$$

$$0 - 1 = a(x+1)^2 \Longrightarrow (x+1)^2 = \frac{-1}{a}$$

$$r = \sqrt{\frac{-1}{a}} \qquad x = -1 \pm r$$

$$\alpha = -1 + r \qquad \beta = -1 - r$$

$$(\alpha^2 + \beta^2) = 5 \longrightarrow (-1+r)^2 + (-1-r)^2 =$$

$$(1 - 2r + r^2) + (1 + 2r + r^2) = 2 + 2r^2$$

$$2r^2 + 2 = 5 \longrightarrow r^2 = \frac{3}{2}$$

$$\frac{-1}{a} = \frac{3}{2} \longrightarrow a = -\frac{2}{3}$$

$$y - 1 = a(0+1)^2 = a \Longrightarrow \qquad \longleftarrow x = 0 \longleftarrow \text{عرض}$$

$$y = 1 + a = 1 - \frac{2}{3} = \frac{1}{3} \longrightarrow \boxed{\text{عرض از مبدا} = \tfrac{1}{3}}$$