(۱

$\frac{L}{w}=\frac{5}{4} \Rightarrow 5w=4L$, $\boxed{w=\frac{4}{5}L}$ $\qquad S_1=L\cdot w=L\cdot(\frac{4}{5}L)=\frac{4}{5}L^2$

$\varphi=\frac{1+\sqrt5}{2}$ $\qquad$ طول جدید $=L'$ $\qquad \frac{L'}{w}=\varphi \to L'=\varphi\cdot w=\varphi\cdot(\frac{4}{5}L)$ $\qquad$ ۲

$A_2=L'\cdot w=(\frac{4}{5}L\cdot\varphi)(\frac{4}{5}L)=\frac{16}{25}\varphi L^2$ $\qquad \frac{A_2}{A_1}=\frac{\frac{16}{25}\varphi L^2}{\frac{4}{5}L^2}=\frac{5\times16\varphi}{4\times25}$

$\Rightarrow \frac{4}{5}\varphi=\frac{4}{5}\times\frac{1+\sqrt5}{2}=\frac{2+2\sqrt5}{5}$

(۲

طول مستطیل $=a$, عرض $=b$, $b>a$

$\frac{\text{قطر}}{\text{طول}}=\varphi=\frac{1+\sqrt5}{2} \to$ قطر $=\sqrt{a^2+b^2} \to \frac{\sqrt{a^2+b^2}}{a}=\varphi \xrightarrow{\text{توان ۲}} a^2+b^2=\varphi^2a^2 \to$

$b^2=\varphi^2a^2-a^2=(\varphi^2-1)a^2 \quad (1+\varphi=\varphi^2) \to \varphi^2-1=(\varphi+1)-1=\varphi \to b^2=\varphi a^2$

$\frac{a}{b}=r \quad \xrightarrow{a^2\varphi=b^2} b=\sqrt\varphi\, a \ (a>0, b>0) \to \frac{a}{b}=\frac{1}{\sqrt\varphi} \to r=\frac{1}{\sqrt\varphi} \to r^2=(\frac{1}{\sqrt\varphi})^2=\frac{1}{\varphi}$ $\qquad$ ۲

$\frac{1}{\varphi}=\frac{\sqrt5-1}{2}=1-\varphi=\frac{1}{\varphi} \to r^2=\frac{\sqrt5-1}{2}$

(۳

$(\sqrt{2a^2+4a})^2=(2-3a)^2 \Rightarrow 2a^2+4a=4-12a+9a^2$ $\qquad 7a^2-16a+4=0 \xrightarrow{\times} a^2-16a+28=0$

$(a-14)(a-2)=0 \to a=\frac{14}{7}$ ✗ , $a=\frac{2}{7}$ ✓ $\qquad \frac{a+1}{a}=\frac{\frac{2}{7}+1}{\frac{2}{7}}=4,5]$ $\qquad$ ۲

(۴

$\frac{\sqrt{n+1}}{\sqrt{n-1}+3}-\frac{\sqrt{n+1}}{3-\sqrt{n-1}}=\frac{n-1}{\sqrt{n-1}} \Rightarrow \frac{(\sqrt{n+1})(3-\sqrt{n-1})-(\sqrt{n+1})(\sqrt{n-1}+3)}{3(\sqrt{n-1})(-\sqrt{n-1})}$

$\sqrt{n+1}\left(\frac{-2\sqrt{n-1}}{9-n+1}\right)=\frac{(\sqrt{n-1})^2}{\sqrt{n-1}} \to 2\sqrt{n+1}=n-10$

$\Rightarrow \frac{(3-\sqrt{n-1})(\sqrt{n+1})-(3+\sqrt{n-1})(\sqrt{n+1})}{10-n}=(\sqrt{n+1})(3-\sqrt{n-1})-3-\sqrt{n-1}$ $\qquad$ ۰/۵

$\to n^2-24n+96 \to n=12\pm4\sqrt3$

$\Rightarrow (\sqrt{n+1})(\cancel{3}-\sqrt{n-1}-\cancel{3}-\sqrt{n-1})=-2\sqrt{n+1}-2\sqrt{n-1}$ $\quad \frac{}{10-n}=\frac{n-1}{\sqrt{n-1}} \Rightarrow \frac{-2\sqrt{n+1}}{10-n}=\sqrt{n+1}$

$(\sqrt n)(12-4\sqrt3)(\sqrt n+1)$ $\quad n\geq10$ و معادله یک ریشه دارد

$\Rightarrow (10-n)(\cancel{\sqrt{n+1}})=-2(\cancel{\sqrt{n+1}}) \Rightarrow 10-n=-2 \Rightarrow n=12$

(۵

$\frac{1}{\sqrt{2-n}+2}-\frac{1}{2-\sqrt{2-n}}=\frac{2-n}{5\sqrt{2-n}} \Rightarrow \frac{(2-\sqrt{2-n})-(\sqrt{2-n}+2)}{4-(2-n)} \Rightarrow \frac{-4\sqrt{2-n}}{2(-2\sqrt{2-n})}\Big/ 2+n=\frac{2-n}{5\sqrt{2-n}}$ $\qquad$ ۰/۵

$4n^2=-20(2-n) \Rightarrow 4-n^2=-40+20n$

$\frac{1}{\sqrt{2-n}+2}-\frac{1}{2-\sqrt{2-n}}=\frac{-2\sqrt{2-n}}{n+2} \to \frac{-2\sqrt{2-n}}{n+2}=\frac{2-n}{5\sqrt{2-n}}$

$n^2+20n-44=0$

$(n+22)(n-2)$ $\quad n=-22$, $n=2$

$\to n^2+10n-24=0$ $\begin{cases} n=2 \to \text{مخرج صفر است} \\ n=-12 \to \text{ریشه است} \end{cases}$

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(6

$\frac{1}{x^2}+\frac{1}{(1-x)^2}=\frac{160}{9}$ $x^2=a^2$ $(1-x)^2=b^2$

0,5

$\frac{a^2+b^2}{a^2b^2}=\frac{(a+b)^2-2ab}{a^2b^2}=\frac{x+1-x-2(x)(1-x)}{a^2b^2}=\frac{1-2x+2x^2}{a^2b^2}=\frac{2x^2-2x+1}{a^2b^2}$

$=\frac{x^2-2x+2}{\cancel{a^2b^2}}$ $a^2(x-1)^2$ $=\frac{x^2-2x+2}{x^2(1-2x+x^2)}=\frac{2x^2-2x+1}{x^2(x^2-2x+1)}$

$\frac{2(x^2-x)+1}{(x-x^2)^2}=\frac{160}{9}$ $\frac{2t+1}{t^2}=\frac{160}{9}$

$\to t=0{,}3,\ \frac{-3}{16}$

$S_1=1$ $S_2=1$ $1+1=2$

(7

$-x^3+4x^2+25x-100\geq 0$ $(x^2-25)(4-x)\geq 0$ $x^2=25 \Rightarrow x=\pm 5$

$x=4$

-5		4		5	
$+$		$-$		$+$	$-$

$[-\infty, -5]\cup[4,5]$

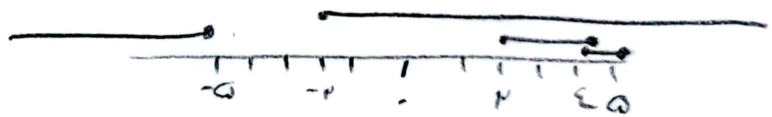
5

$-x^2-6x-8\geq 0 \to -(x^2-6x+8)\geq 0$ $x^2-6x+8\leq 0 \to (x-4)(x-2)=0$

$x=2,4$

2		4	
$-$		$-$	$+$

$[2,4]$



جواب $=4$

۸)

$y=|x+2|+|x-1|$, $3y+x=17$ $\quad y=-\frac{1}{3}x+\frac{17}{3}$

۰٫۵

$-\frac{1}{3}x+\frac{17}{3}=|x+2|+|x-1|$

$|x+2|+|x-1|+\frac{1}{3}=\frac{17}{3}$

① $x\le -2 \rightarrow -x-2-x+1+\frac{1}{3}x=\frac{17}{3} \rightarrow -\frac{7}{3}x=\frac{14}{3} \rightarrow x=-2$ ✓

② $-2\le x<1 \rightarrow x+2-x+1+\frac{1}{3}x=\frac{17}{3} \rightarrow x=\frac{17}{10}=+1.7$ غ ق ق

③ $x\ge 1 \rightarrow x+2+x-1+\frac{1}{3}x=\frac{17}{3} \rightarrow x=2$ ق ق

$-2 \qquad 1$

$-6x-3=17-x$ $(-4,7)$ | $x=8$ X | $6x+3=17-x$ $(2,5)$

$AB=\sqrt{(-4-2)^2+(7-5)^2}=2\sqrt{10}$

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۹)

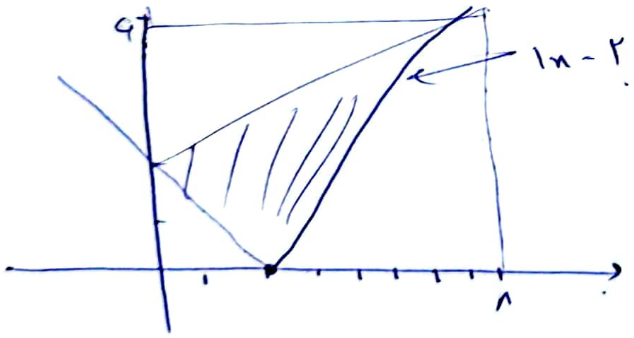
$y=\sqrt{x^2-4x+4}=\sqrt{(x-2)^2}=|x-2| \quad x\ge 2$

$y=\frac{1}{2}x+2 \rightarrow \frac{1}{2}x+2=|x-2| \rightarrow$

۱٫۵

$x-2=\pm(\frac{1}{2}x+2)$

$\begin{cases} x-2=\frac{1}{2}x+2 \rightarrow x=8 & (8,6) \\ x-2=-\frac{1}{2}x-2 \rightarrow \frac{3}{2}x=0 \rightarrow x=0 & (0,2) \end{cases}$



$|x-2|=\frac{1}{2}x+2$

$A(0,2)\ B(2,0)\ C(8,6)$

$AB=2\sqrt{2}$ $\quad S=\frac{2\sqrt{2}\times 6\sqrt{2}}{2}=12$

$BC=6\sqrt{2}$

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۱۰)

$\frac{1}{x}+\frac{1}{x-9}=\frac{1}{20}$ $\qquad$ فرهاد $x-9$ ، x بهروز

۲

$\frac{\overset{2x-9}{\cancel{x-9+x}}}{x^2-9x}=\frac{1}{20} \rightarrow x^2-9x=40x-180 \rightarrow x^2-31x-180=0$

$(x-36)(x-5)=0 \rightarrow (x-36)(x-5)=0$

$\begin{cases} x=36 ✓ \\ x=5 \end{cases}$ غ ق ق

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