

$$\frac{1+\sqrt{a}}{r} \leadsto \frac{1+\sqrt{a}}{r} = \frac{a+y}{r} \rightarrow \frac{1+\sqrt{a}}{r} \times \frac{r}{a} = \frac{r}{a} \times \frac{1+\sqrt{a}}{r} = \frac{1+\sqrt{a}}{a}$$

$$3a + \sqrt{2a^2 + 4a} = r \rightarrow \sqrt{2a^2 + 4a} = r - 3a \Rightarrow 2a^2 + 4a = (r - 3a)^2 \Rightarrow 2a^2 + 4a = r^2 - 6ar + 9a^2 \Rightarrow 7a^2 - 6ar + r^2 = 0$$

پس $r - 3a > 0 \Rightarrow a \leq \frac{r}{3}$

$$\frac{a+1}{a} = ? \rightarrow \frac{9}{r} \Rightarrow 7a^2 - 14a + 4 = 0 \rightarrow a = \frac{14 \pm \sqrt{14^2 - 4 \cdot 7 \cdot 4}}{14} = \frac{14 \pm 12}{14} \Rightarrow a = 2 \text{ یا } a = \frac{2}{7}$$

چون $r - 3a > 0$ پس $a = \frac{2}{7}$ $\rightarrow \frac{r}{7} + 1 = \frac{9}{7} \Rightarrow \frac{r}{7} = \frac{9}{7} - 1 = \frac{2}{7} \Rightarrow r = 2$

حل = L
عرض = w $\rightarrow \frac{\sqrt{L^2 + w^2}}{L} = \frac{1+\sqrt{a}}{r} \rightarrow \frac{L^2 + w^2}{L^2} = \left(\frac{1+\sqrt{a}}{r} \right)^2$

$$\Rightarrow 1 + \left(\frac{w}{L} \right)^2 = \left(\frac{1+\sqrt{a}}{r} \right)^2 \rightarrow \left(\frac{w}{L} \right)^2 = \frac{1+\sqrt{a}}{r} \rightarrow \frac{1}{\left(\frac{w}{L} \right)^2} = \frac{1}{\frac{1+\sqrt{a}}{r}} \Rightarrow \frac{L^2}{w^2} = \frac{r}{1+\sqrt{a}} \Rightarrow \frac{1}{\frac{1+\sqrt{a}}{r}} = \frac{1+\sqrt{a}}{r} - 1 \Rightarrow \frac{1+\sqrt{a}}{r} - 1 = \frac{\sqrt{a}-1}{r}$$

$$\frac{\sqrt{x+1}}{\sqrt{x-1}+3} - \frac{\sqrt{x+1}}{3-\sqrt{x-1}} = \frac{x-1}{\sqrt{x-1}} = t$$

داریم $\frac{1}{t+3} - \frac{1}{3-t} = \sqrt{t^2+2} \times \frac{-2t}{9-t^2} = \frac{-2t\sqrt{t^2+2}}{9-t^2} = t \xrightarrow{t=0} \frac{-2\sqrt{2}}{9} = 1$

$\Rightarrow 2\sqrt{t^2+2} = t^2 - 9 \xrightarrow{t^2 \geq 9, y=t^2} 2(y+2) = (y-9)^2 \Rightarrow y^2 - 22y + 17 = 0 \Rightarrow (11+\sqrt{44}) + 1 = 12+\sqrt{44}$

$$\frac{1}{\sqrt{r-x}+r} - \frac{1}{r-\sqrt{r-x}} = \frac{r-x}{\sqrt{r-x}} = t \rightarrow \frac{r-t}{t^2-r} = \frac{t}{a} \xrightarrow{t=0} \frac{r}{-r} = -1 \Rightarrow \frac{r}{t^2-r} = -\frac{1}{a} \Rightarrow t^2 = 1 \Rightarrow t = \pm 1$$

$\Rightarrow x = r - a^2 = r - 1^2 = r - 1 = -12$ غلط

پس $x = -12$ یا $x = 2$ $\rightarrow$ پاسخ

$$\frac{1}{x^2} + \frac{1}{(1-x)^2} = \frac{14}{9}$$

$$\left(\frac{1}{x} + \frac{1}{1-x}\right)^2 - 2\left(\frac{1}{x(1-x)}\right) = \frac{14}{9} \rightarrow \left(\frac{1}{x(1-x)}\right)^2 - 2\left(\frac{1}{x(1-x)}\right) = \frac{14}{9} - \frac{4}{9} = \frac{10}{9}$$

$$t^2 - 2t = \frac{10}{9} \rightarrow t^2 - 2t + 1 = \frac{10}{9} + 1 \rightarrow (t-1)^2 = \frac{19}{9} \rightarrow t-1 = \pm \frac{\sqrt{19}}{3}$$

$$\rightarrow t-1 = \frac{\sqrt{19}}{3} \rightarrow t = \frac{1+\sqrt{19}}{3} \rightarrow \frac{1}{x(1-x)} = \frac{1+\sqrt{19}}{3} \rightarrow \frac{1}{x} - \frac{1}{1-x} = \frac{1+\sqrt{19}}{3}$$

$$\rightarrow t-1 = -\frac{\sqrt{19}}{3} \rightarrow t = \frac{1-\sqrt{19}}{3} \rightarrow \frac{1}{x(1-x)} = \frac{1-\sqrt{19}}{3} \rightarrow \frac{1}{x} - \frac{1}{1-x} = \frac{1-\sqrt{19}}{3}$$

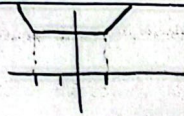
$$\sqrt{x} + \sqrt{-x^2 + 4x - 1} + \sqrt{x^2 + \sqrt{-x^2 + 4x - 1}} = x + 2$$

$$-x^2 + 4x - 1 \geq 0 \quad x^2 + 4x + 1 \leq 0 \rightarrow (x-2)(x-4) \leq 0 \rightarrow 2 \leq x \leq 4$$

$$x^2 - 4x^2 - 4x + 1 \leq 0 \rightarrow -3x^2 - 4x + 1 \leq 0 \rightarrow 3x^2 + 4x - 1 \geq 0 \rightarrow (3x-1)(x+1) \geq 0 \rightarrow x \geq \frac{1}{3} \text{ or } x \leq -1$$

ملاحظات: $2 \leq x \leq 4$ $\rightarrow$ $x=2$ $\rightarrow \sqrt{2} + \sqrt{14} = 2+2=4$ $\rightarrow$ $x=4$ $\rightarrow$ $\sqrt{4} + \sqrt{12} = 2+2=4$

$$y = 1x + 2 + 1x - 1 \rightarrow y + x = 1$$



$$AB \text{ دایره } \Rightarrow \sqrt{(x+2)^2 + (y-1)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$$

$$\Rightarrow ① = (x+2) + (x-1) = 2x+1 \rightarrow 3(2x+1) + x = 17 \rightarrow 6x+3+x=17 \rightarrow 7x=14 \rightarrow x=2$$

$$\Rightarrow ② = -2 \leq x \leq 1 \rightarrow y = (x+2) + (1-x) = 3 \rightarrow 3(3) + x = 17 \rightarrow 9+x=17 \rightarrow x=8$$

$$\Rightarrow ③ = x \leq -2 \rightarrow y = -(x+2) + (1-x) = -2x-1 \rightarrow 3(-2x-1) + x = 17 \rightarrow -6x-3+x=17 \rightarrow -5x=20 \rightarrow x=-4$$

$$y = \sqrt{x^2 - 4x + 4} = |x-2| \quad \begin{matrix} \text{IF } x \geq 2 \rightarrow x-2 \rightarrow \frac{1}{x}x+2 = x-2 \rightarrow x=4 \\ \text{IF } x < 2 \rightarrow \frac{1}{x}x+2 = 2-x \rightarrow x=0 \end{matrix}$$

$$y = \frac{1}{x}x + 2$$

$$\text{مساحت دایره} = \frac{4+4}{4} \times \pi = 2\pi$$

$$\Rightarrow \text{مساحت کل} = 4\pi - 2\pi = 2\pi$$

$$\rightarrow 2\pi \rightarrow \frac{2\pi}{4} = \frac{1}{2}\pi \rightarrow \frac{1}{2}\pi \times 2 = \pi$$

$$\rightarrow 4\pi \rightarrow \frac{4\pi}{4} = \pi \rightarrow \pi \times 2 = 2\pi$$

$$\frac{1}{B} + \frac{1}{F} = \frac{1}{f} \Rightarrow \frac{1}{B} + \frac{1}{B+4} = \frac{1}{f} \rightarrow \frac{2B+4}{B(B+4)} = \frac{1}{f} \rightarrow f \cdot (2B+4) = B^2+4B$$

$$\Rightarrow f \cdot B + 4f = B^2 + 4B \rightarrow B^2 - 2fB - 4f = 0 \rightarrow B = \frac{2f \pm \sqrt{4f^2 + 16f}}{2}$$

$$\Rightarrow \frac{2f+4f}{2} = 3f \rightarrow \text{در صورتی که } f=1 \rightarrow 3$$