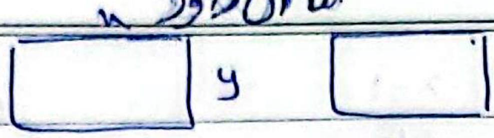
آل داود

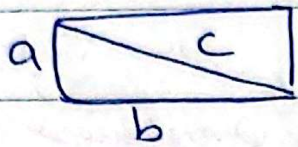


$\frac{x}{y} = \frac{5}{4}$

(1) طول $\frac{x+t}{y} = \frac{1+\sqrt{5}}{2}$ عرض

$\frac{(x+t)y}{xy} = \frac{1+\sqrt{5}}{2} \times \frac{4}{5} \to 0/4(1+\sqrt{5})$

(2)



$c^2 = a^2+b^2$ $\qquad \frac{c}{b} = \frac{1+\sqrt{5}}{2}$

$c = \sqrt{a^2+b^2}$ $\qquad \frac{\sqrt{a^2+b^2}}{b} = \frac{1+\sqrt{5}}{2}$

$\sqrt{1+(\frac{a}{b})^2} = \frac{1+\sqrt{5}}{2} \xrightarrow{\text{به توان ۲}} 1+(\frac{a}{b})^2 = \frac{\cancel{1+5}+2\sqrt{5}}{4} \to (\frac{a}{b})^2 = \frac{6+2\sqrt{5}}{4}-1$

$(\frac{b}{a})^2 = \frac{1}{1/6} = \frac{10}{16} \leftarrow \frac{1}{\frac{1+\sqrt{5}}{2}-1} = (\frac{b}{a})^2$

(3) $(\sqrt{2a^2+4a})^2 = (2-3a)^2 \to 2a^2+4a = 4+9a^2-12a$

$7a^2-16a+28=0 \to (a-14)(a-2)=0$

$a = \cancel{\frac{14}{7}}, \frac{2}{7}$

$\sqrt{2(2)^2+4a} \neq \cancel{2-6}$ ← غیرقابل قبول (دو معادله صدق نمی‌کند)

← قابل قبول (در معادله صدق می‌کند)

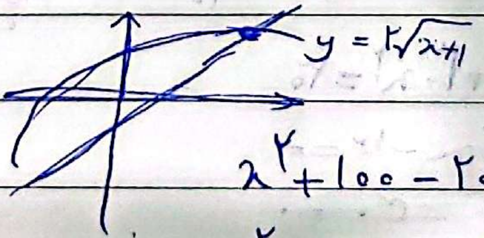
$\sqrt{2(\frac{2}{7})^2+4(\frac{2}{7})} = 2-3(\frac{2}{7})$ ✓

$\frac{\frac{2}{7}+1}{\frac{2}{7}} = \frac{\frac{9}{7}}{\frac{2}{7}} = \frac{9}{2} = 4/5$

(4) حل معادله

$\frac{\sqrt{x+1}}{\sqrt{x-1}+3} - \frac{\sqrt{x+1}}{3-\sqrt{x-1}} = \frac{x-1}{\sqrt{x-1}} \xrightarrow{x \neq 1}$

$\frac{\sqrt{x+1}(3-\sqrt{x-1}-\sqrt{x-1}-3)}{(3)^2-(x-1)} = \sqrt{x-1} \to \frac{-2\sqrt{x-1}\sqrt{x+1}}{10-x} = \sqrt{x-1}$



$x-10 = 2\sqrt{x+1} \leftarrow x \neq 0$

$x^2+100-20x = 4x+4$ یک جواب دارد.

$x^2-24x+96=0$

$x = 12+4\sqrt{3}$

(5)

$$\frac{2-\sqrt{2-x}-\sqrt{2-x}-2}{4-(2-x)}=\frac{2\sqrt{2-x}}{(x-2)^2}=\frac{x-2}{-5\sqrt{2-x}}$$

$$(x-2)^2=-10(2-x)\longrightarrow 10x-20=x^2+4-4x$$

$$x^2-14x+24=0\quad (x-2)(x-12)=0$$

$$x=2,12$$

در ریشه پیشت طرح!

هر دو در معادله صدق می‌کنند.

(6)

$$\left(\frac{1}{x}+\frac{1}{1-x}\right)^2-2\frac{1}{x(1+x)}=\frac{160}{9}$$

$$\left(\frac{1}{x(1-x)}\right)=t$$

$$\frac{1}{x(1-x)^2}-2\left(\frac{1}{x(1-x)}\right)=\frac{160}{9}$$

$$t^2-2t=\frac{160}{9}\rightsquigarrow t^2-2t+1=\frac{169}{9}$$

$$(t-1)^2=\frac{169}{9}\rightsquigarrow t-1=\frac{13}{3}\rightarrow t=\frac{16}{3}$$

$$t-1=\frac{-13}{3}\rightarrow t=-\frac{10}{3}$$

$$\frac{1}{x(1-x)}=\frac{16}{3}\rightsquigarrow -x^2+x=\frac{3}{16}\rightsquigarrow x(1-x)=\frac{3}{4}$$

$$16x-16x^2=3\rightsquigarrow x^2-16x+48=0\quad (x-4)(x-12)=0$$

$$x=\frac{4}{16},\frac{12}{16}$$

$$-\frac{1}{4},\frac{3}{4}$$

$$-\frac{1}{4}+\frac{3}{4}=1$$

$$S_1=1$$

$$\frac{1}{x(1-x)}=-\frac{10}{3}\rightsquigarrow$$

$$S_1+S_2=2$$

$$-10x+10x^2=3$$

$$10x^2-10x-3=0$$

$$x^2-10x-30=0\quad \xrightarrow{\Delta>0} x_1+x_2\qquad \frac{\cancel{5+\sqrt{55}}+\cancel{5-\sqrt{55}}}{10}=1\Big\}=S_2$$

(7)

$-x^2+6x-8 \geq 0$

$x+2 \geq 0$
$x \geq -2$

$-(x^2-6x+8) \geq 0$

$(x-2)(x-4) \leq 0$

$\begin{array}{ccccc} & 2 & & 4 & \\ + & | & - & | & + \end{array} \longrightarrow 2 \leq x \leq 4$

$-x^2(x-4)+25(x-4) \geq 0$

$(x-4)(-x^2+25) \geq 0$

$(x-4)(5-x)(5+x) \geq 0 \xrightarrow{\text{تعیین علامت}} x \leq -5 \ \cup \ 4 \leq x \leq 5$

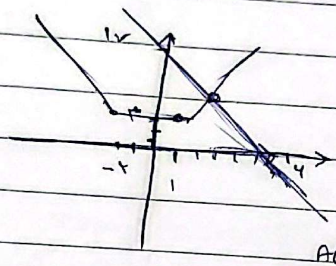
$\xrightarrow[\text{با شرط}]{\text{اشتراک}} x = 4 \longrightarrow$ در هر دو معادله صدق می‌کند! $\longrightarrow$ جواب طرفین

(8)

$\begin{cases} 2x+1 & x>1 \\ 3 & -2<x\leq 1 \\ -2x-1 & x\leq -2 \end{cases}$

$\begin{array}{c|c} 0 & 17 \\ \hline \end{array} \quad \begin{array}{c|c} 0 & 5.6 \end{array}$

$3y + x = 17$



$2x+1 = \frac{-x+17}{3} \longrightarrow 6x+3 = -x+17 \longrightarrow x=2 \quad A(2,5)$

$-2x-1 = \frac{-x+17}{3} \longrightarrow -6x-3 = -x+17 \longrightarrow x=-4 \quad B(-4,7)$

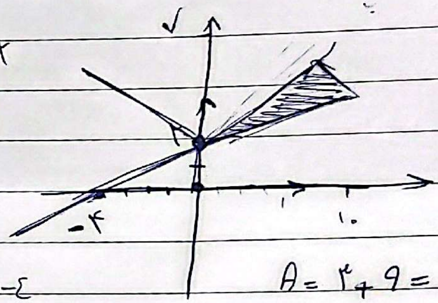
$AB = \sqrt{(2-(-4))^2+(5-7)^2} = 2\sqrt{10}$

(9)

$y = \sqrt{x^2-4x+4} \Rightarrow \sqrt{(x-2)^2} = |x-2|$

$y = \frac{1}{2}x+2$

$\begin{array}{c|c} 0 & \\ 2 & 0 \end{array}$



$0 = \frac{1}{2}x+2$

$-2 = \frac{1}{2}x$

$x = \frac{-2}{\frac{1}{2}} = -4$

$A = 3+9 = 12$

$|x-2| = \frac{1}{2}x+2$

$x-2 = \frac{1}{2}x+2 \longrightarrow \frac{1}{2}x = 4 \quad x = 8$

$\cancel{-x+2} = \cancel{-2}-\frac{1}{2}x \longrightarrow x = 0$

① مساحت بخش اول (از ۰ تا ۲): ۳ واحد مربع

② مساحت بخش دوم (از ۲ تا ۸): ۹ واحد مربع

(10)

	کل	یک ساعت
فرهاد	$x \longrightarrow$	$\frac{1}{x}$
بهروز	$x-9 \longrightarrow$	$\frac{1}{x-9}$

$\times (x)(x-9)(20)$

$\frac{1}{x}+\frac{1}{x-9} = \frac{1}{20}$

$20x-180+20x = x^2-9x$

$0 = x^2-49x+180$

$(x-4)(x-45)=0$

$x = 4, 45$

$\xrightarrow{x=45} \boxed{x-9=36}$ قبول

$\xrightarrow{x=4} x-9 = -5$ غیر قابل قبول

PASARGAD [illegible]

DATE : ____________