۲۰

مطهره میرمحمدی ←

۱ ←

$$\frac{L}{W} = \frac{\sqrt{5}+1}{2}$$

$5x$ ، $4x$

$$\to S_{\text{اولیه}} = 20x^2$$

L ، $4x$

$$\to \frac{L}{\cancel{2}\times 4x} = \frac{\sqrt{5}+1}{\cancel{2}} \to$$

$$L = 4x(\sqrt{5}+1) \longrightarrow S_{\text{جدید}} = 4x \times 4x(\sqrt{5}+1)$$

$$\frac{\cancel{4x}\times \overset{0/8}{\cancel{4x}}(\sqrt{5}+1)}{\cancel{4x}\times \cancel{5x}} = \boxed{0/8(\sqrt{5}+1)}$$

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۲ ←

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y ، x ، $\sqrt{x^2+y^2}$

$$\to \frac{\sqrt{x^2+y^2}}{y}$$

$$y^2/x^2 = ?$$

$$\frac{\sqrt{x^2+y^2}}{y} = \frac{\sqrt{5}+1}{2} \xrightarrow[2]{\text{توان}} \frac{x^2+y^2}{y^2} = \frac{\overset{3}{\cancel{6}}+\cancel{2}\sqrt{5}}{\underset{2}{\cancel{4}}}$$

$$\longrightarrow 2x^2+2y^2 = 3y^2 + y^2\sqrt{5} \longrightarrow 2x^2 = y^2 + y^2\sqrt{5}$$

$$\longrightarrow x^2 = y^2\left(\frac{1+\sqrt{5}}{2}\right) \longrightarrow \boxed{\frac{y^2}{x^2} = \frac{\sqrt{5}-1}{2}}$$

۳ ←

$$3a+\sqrt{2a^2+4a}=2$$

$$\sqrt{2a^2+4a}=2-3a \xrightarrow{\text{۲ توان}}$$

$$2a^2+4a=9a^2+4-12a \longrightarrow$$

$$7a^2+4-16a=0 \longrightarrow \Delta=144$$

$$a=\frac{16\pm12}{14} \longrightarrow a=2,\ \frac{2}{7} \longrightarrow a=2/7$$

$$\frac{\cancel{\frac{2}{7}+\frac{5}{7}}\ \ \frac{9}{4}}{\frac{2}{4}}=9/2=\boxed{4.5}$$

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۴ ←

$$\underbrace{\frac{\sqrt{x+1}}{\sqrt{x-1}+3}-\frac{\sqrt{x+1}}{3-\sqrt{x-1}}}=\frac{x-1}{\sqrt{x-1}}$$

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$$\hookrightarrow \frac{3\cancel{\sqrt{x+1}}-\sqrt{x^2-1}-\sqrt{x^2-1}-\cancel{3\sqrt{x+1}}}{\underbrace{9-(x-1)}_{10-x}}$$

$\sqrt{x+1}\times\sqrt{x-1}$

$$\frac{-2\sqrt{x^2-1}}{10-x}=\frac{x-1}{\sqrt{x-1}} \longrightarrow -2\sqrt{x+1}\times(x-1)=(x-1)(10-x)$$

$x>1 \Leftarrow x-1>0$

$x-1$ توانی

$$-2\sqrt{x+1} = 10 - x \rightarrow x>10$$

$$\rightarrow 4(x+1) = x^2 + 100 - 20x \rightarrow$$

$$\underbrace{4(x+1)}_{4x+4}$$

$$x^2 - 24x + 96 = 0 \rightarrow \Delta = 8\sqrt{3}$$

$$x = \frac{24 \pm 8\sqrt{3}}{2} = 12 \pm 4\sqrt{3} \rightarrow \boxed{12 + 4\sqrt{3}}$$

این قابل قبول چون $x \geq 10$ باید باشه

$$\frac{1}{\sqrt{2-x}+2} - \frac{1}{2-\sqrt{2-x}} = \frac{2-x}{5\sqrt{2-x}} \leftarrow 5$$

2

$$\sqrt{2-x} = t$$

$$\rightarrow \frac{1}{t+2} - \frac{1}{2-t} = \frac{2-t-t-2}{4-t^2} = \frac{-2t}{4-t^2}$$

$$\frac{2-x}{5\sqrt{2-x}} = \frac{t^2}{5t} = \frac{t}{5}$$

$$\frac{-2\cancel{t}}{4-t^2} = \frac{\cancel{t}}{5} \rightarrow -10 = 4 - t^2 \rightarrow$$

$$t^2 \rightarrow 2-x$$

$$-10 = x + 2 \rightarrow x = -12 \rightarrow$$

این معادله ریشه + ندارد

5

$$\frac{1}{x^2}+\frac{1}{(1-x)^2}=\frac{160}{9}$$

$$x=\frac{1}{2}+t$$

$$\frac{1}{\left(\frac{1}{2}+t\right)^2}+\frac{1}{\left(\frac{1}{2}-t\right)^2}=\frac{160}{9}$$

$$\frac{\frac{1}{4}+t^2-\cancel{t}+\frac{1}{4}+t^2\cancel{+t}}{\left(\frac{1}{4}-t^2\right)^2}$$

$$\frac{\cancel{2}\left(\frac{1}{4}+t^2\right)}{\left(\frac{1}{4}-t^2\right)^2}=\frac{\cancel{160}^{\,80}}{9}\longrightarrow \frac{9}{4}+9t^2=$$

$$t^4+\frac{1}{16}-\frac{t^2}{2}$$

$$320t^4-196t^2+11=0\longrightarrow \Delta=156$$

$$t^2=a=\frac{196\pm156}{640}\longrightarrow a=\frac{1}{16},\ \frac{11}{20}\longrightarrow$$

$$t=\pm\frac{1}{4},\ \pm\frac{\sqrt{55}}{10}\longrightarrow x=\frac{1}{2}+t\Rightarrow x=\frac{1}{4},\ \frac{3}{4},\ \frac{1}{2}+\frac{\sqrt{55}}{10},\ \frac{1}{2}-\frac{\sqrt{55}}{10}$$

$$\frac{1}{4}+\frac{3}{4}+\left(\frac{1}{2}\pm\frac{\sqrt{55}}{10}\right)=\boxed{2}$$

۷ ← ۲

$$-x^3+4x^2+25x-100=-(x^3-4x^2-25x+100)$$

$$=-(x-4)(x^2-25)=-(x-4)(x-5)(x+5)\geqslant 0$$

$$(x-4)(x-5)(x+5)\leqslant 0 \Rightarrow \begin{array}{c|c|c|c} & -5 & 4 & 5 \\ \hline - & + & - & + \end{array}$$

$$x\in(-\infty,-5]\cup[4,5]$$

$$-x^2+6x-8=-(x-2)(x-4)\geqslant 0 \longrightarrow$$

$$(x-2)(x-4)\leqslant 0 \rightarrow \begin{array}{c|c|c} & 2 & 4 \\ \hline + & - & + \end{array}$$

$$\rightarrow x\in[2,4]$$

اشتراک دامنه $\longrightarrow \big((-\infty,-5]\cup[4,5]\big)\cap[2,4]=$

$$\{4\}$$

$$\xrightarrow{x=4}\sqrt{4+\sqrt{0}}+\sqrt{16+\sqrt{0}}=2+4=6$$

معادله فقط یک جواب دارد ← $x=4$

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۸ ← ۲

$$y=|x+2|+|x-1| \rightarrow \begin{cases} 2x+1 & x\geqslant 1 \\ 3 & -2\leqslant x\leqslant 1 \\ -2x-1 & x\leqslant -2 \end{cases}$$

$$y=\frac{17-x}{3}$$

$x \leq -2 \longrightarrow -2x-1 = \dfrac{17-x}{3} \longrightarrow$

$-6x-3 = 17-x \longrightarrow -5x = 20 \longrightarrow x = -4$

$\longrightarrow y = 7$

$A(-4, 7)$

$2x+1 = \dfrac{17-x}{3} \longrightarrow 6x+3 = 17-x \longrightarrow 7x = 14 \longrightarrow$

$x = 2 \longrightarrow$

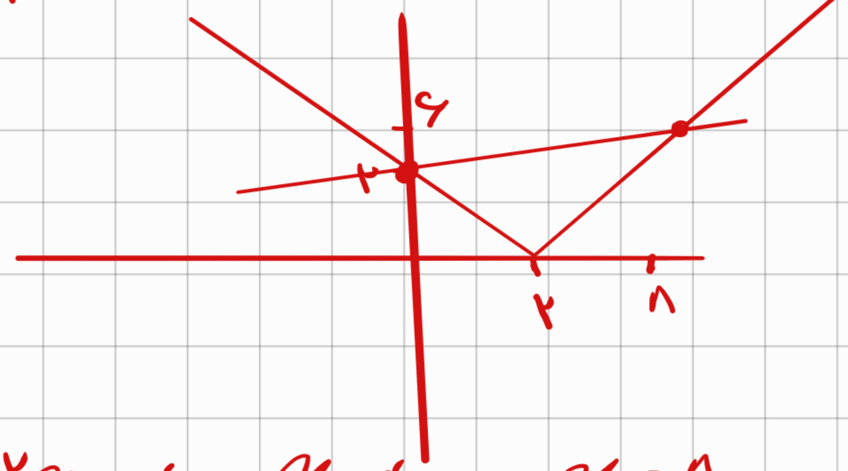
$y = 5$

$B(2, 5)$

$\overline{AB} = \sqrt{\underbrace{(2+4)^2+(5-7)^2}_{40}} = \boxed{2\sqrt{10}}$

$y = \sqrt{\underbrace{x^2-4x+4}_{(x-2)^2}} = |x-2|$ 　　 ۲ $\longleftarrow$ ۹

$y = \frac{1}{2}x+2$



$x \geq 2$

$x-2 = \frac{1}{2}x+2 \longrightarrow 2x-4 = x+4 \longrightarrow x = 8 \longrightarrow$

$y = 6$

$2-x = \frac{1}{2}x+2 \longrightarrow 4-2x = x+4 \longrightarrow x=0, \; y=2$

$$\begin{array}{ccc} ۲ & ۰ & ۱ \\ ۰ & ۲ & ۱ \\ ۸ & ۶ & ۱ \end{array} \longrightarrow \frac{۱}{۲}\left(\underbrace{۱۶+۱۲+۰-۴-۰-۰}_{۲۴}\right)$$

$$S_D = \frac{۱}{۲} \times ۲۴ = \boxed{۱۲}$$

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۵ ← ۱۵

$$\left.\begin{array}{l} \frac{۱}{B} + \frac{۱}{F} = \frac{۱}{۲۰} \\ B + ۹ = F \end{array}\right\} \longrightarrow \frac{۱}{B} + \frac{۱}{B+۹} = \frac{۱}{۲۰}$$

$$\frac{\overset{۲B}{\cancel{B+B}}+۹}{B(B+۹)} = \frac{۱}{۲۰} \longrightarrow B^۲ + ۹B = ۴۰B + ۱۸۰$$

$$\longrightarrow B^۲ - ۳۱B - ۱۸۰ = ۰ \longrightarrow \Delta = ۴۱ \longrightarrow$$

$$B = \frac{۳۱ \pm ۴۱}{۲} \longrightarrow \frac{۷۲}{۲} = \boxed{۳۶}$$
